Question for Vogel portion of exam

Please show all work.

Question setup.

- Assume monopolistic competition: each firm i ignores its impact on the Lagrange multiplier on the consumer's budget constraint
- Let c_i denote each consumer's consumption of good i (consumption of i per capita)
- The mass of consumers is L
- Each consumer's utility is given by

$$U = \sum_i u(c_i)$$

  where we assume that $u'(c) > 0$ and $u''(c) < 0$

- Elasticity of demand (w/ monopolistic competition) $\varepsilon \equiv -d \log c_i / d \log p_i$ satisfies the following two properties
  - $\varepsilon'(c) < 0$ (it is both differentiable and decreasing)
  - $\lim_{c \to 0} \varepsilon(c) > 1$
- Production and entry: An unbounded mass of potential entrants chooses whether or not to enter and, if a given entrant sets up a firm it chooses how much to produce. If a potential entrant hires $f + \beta q$ units of labor, it enters and produces q units of output
  - Wordy description to be sure it's clear:
    * There is free entry
    * To enter, a firm must incur a fixed cost of $f > 0$ units of labor
    * After entry, a firm hiring βq additional units of labor produces q units of output, where $\beta > 0$
- Let N denote the number of entrants, c denote consumption per capita of each variety, $x = Lc$ denote each firm's output, and let the wage w be normalized to equal one.

Questions.

1. Does an equilibrium exist? If it exists, is it unique? Prove the answer(s).
2. In what follows, I'm asking about the sign of derivatives. Derivations can be done using either graphs or calculus (although I do not promise graphs will work)
   (a) What is the effect of an increase in β on consumption per capita of each good c?
   (b) What is the effect of an increase in β on each firm's price p?
   (c) What is the effect of an increase in β on the number of entrants N?

Question on Market Size, Competition and Innovation

Consider a market with $L > 0$ identical consumers and a fixed measure of differentiated varieties $i \in [0, M]$. Each consumer solves

$$\max_{\{q_i\}} U \;=\; \sum_{i=0}^{M} \Big(\alpha q_i - \tfrac{\beta}{2} q_i^2 \Big), \qquad \alpha, \beta > 0,$$

subject to the budget constraint

$$\sum_{i=0}^{M} p_i q_i \;=\; I,$$

where I is exogenous income and p_i is the price of variety i. Henceforth let q and p denote the quantity and price of a single representative variety produced by the firm studied below.

1. Show that consumers have the following inverse residual-demand curve for each variety
$$p(q) = \frac{\alpha - \beta q}{\lambda},$$
where $\lambda > 0$ denotes the Lagrange multiplier on the budget constraint. Throughout the exercise, we treat λ as an exogenous parameter summarizing competitive pressure. A higher λ therefore shifts every residual-demand curve downward, which we interpret as stronger competition for a given market size L.

2. A firm with marginal cost c chooses q to maximize total variable profits $L(p - c)q$. Derive the profit-maximizing output q and show that profits per-consumer are:
$$\pi(c; \lambda) = \frac{(\alpha - \lambda c)^2}{4\,\beta\,\lambda}.$$

3. Each firm has a baseline marginal cost $\tilde{c}$. Spending $k \geq 0$ on R&D lowers cost to $c = \tilde{c} - \varepsilon k$ with $\varepsilon > 0$. The R&D expenditure is

$$C_I(k) = c_I k + \frac{\gamma}{2} k^2, \qquad c_I, \gamma > 0.$$

Total profits over all consumers is therefore

$$\Pi(\tilde{c}, k; L, \lambda) = L\,\pi(\tilde{c} - \varepsilon k; \lambda) - C_I(k).$$

Firms choose k to maximize Π. Show that an interior optimum $k(\tilde{c}; L, \lambda)$ satisfies

$$\varepsilon L\, q = \gamma\, k + c_I,$$

where q is the optimal output per consumer for this firm, obtained above.

4. Calculate the derivative (holding λ fixed) $\dfrac{\partial k}{\partial L}$ at an interior optimum and show that it is positive. Show that innovative investment is more responsive for more productive firms: $\dfrac{\partial^2 k}{\partial L\, \partial \tilde{c}} < 0$.

5. Calculate the derivative (holding L fixed) $\dfrac{\partial k}{\partial \lambda}$ and show that it is negative. Show that the reduction in innovation is most pronounced for less productive firms: $\dfrac{\partial^2 k}{\partial \lambda\, \partial \tilde{c}} < 0$.

6. Explain briefly why an export-demand shock that increases both market size L and competition λ can lead high-productivity (low $\tilde{c}$) firms to raise innovation and low-productivity firms to reduce innovation.

7. Briefly outline an empirical design that could test the previous prediction using firm-level data on R&D and export shocks.

1. Consider the Armington model from Anderson and van Wincoop (2003)[1] studied in class. There are N countries. In country n there are L_n workers who produce z_n units of output each. Preferences in country n are:

$$C_n = \left[\sum_{i=1}^{N} \alpha_i^{1/\sigma} C_{ni}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

where C_{ni} is a country n resident's consumption of imports from i . Iceberg trade costs from i to n are τ_{ni}.

   (a) Define an equilibrium.

   (b) Suppose trade costs fall between countries i and j. How does welfare change in country $k \neq i, j$? First discuss intuitively: do you expect country k to be better or worse off? Then, describe a system of equations and data that could be used to measure this welfare change.

   (c) Suppose there are two countries and no trade costs. In the home country, the final good is sold competitively to final consumers and to one firm who monopolizes exports. This firm prices internationally taking into account the foreign demand response. Set up the problem of a home-country planner who may tax or subsidize exports, and discuss whether the planner may have incentives to use these taxes.

2. Consider the setup in Wildasin (1987)[2] studied in class: a measure 1 of households are perfectly mobile across M jurisdictions. n_i households who live in location i derive per capita utility $u(x_i, z_i)$ from a numeraire good x_i and a public good z_i. The numeraire is produced in i using the technology $F_i(n_i, T_i)$ and the public good is produced using the cost function $C_i(n_i, z_i)$ in units of the numeraire. The market allocation is competitive and workers equally own the rents generated by the land endowments T_i.

   (a) Set up the planner's problem. Under what conditions on $C_i(n_i, z_i)$ is the market allocation without policies efficient?

   (b) Suppose the only policy is a head tax τ_i^n paid by residents of i. Under what conditions is this tax sufficient to attain efficiency?

   (c) More generally, what combination of land rent taxes τ_i^L and head taxes τ_i^n are needed to implement efficiency?

   (d) In Fajgelbaum and Gaubert (2020)[3] both amenity and production spillovers are present. Explain how and why the set of instruments that are generically needed to implement optimal policy differs between Fajgelbaum and Gaubert (2020) and Wildasin (1987).

[1] Title: "Gravity with Gravitas: A Solution to the Border Puzzle."
[2] Title: "Theoretical Analysis of Public Economics."
[3] Title: "Optimal Spatial Policies, Geography and Sorting."

3. Briefly explain the procedure used by Donaldson (2018), “Railroads of the Raj” to test the Eaton and Kortum model.

4. Briefly explain the procedure used by Ahlfeldt et al. (2015), “The Economics of Density: Evidence from the Berlin Wall” to demonstrate the relevance of externalities in shaping the city’s responses to the division.

International Economics Field Exam
UCLA, Department of Economics
Questions from Aaron Tornell, Econ 281B
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Consider an economy with moral hazard in credit markets and government bailout guarantees. Concretely, consider a borrower with internal funds w that borrows an ammount b at time t and promises to repay $b*[1+r]$ at time $t+1$. With probability α the borrower will have enough income at time $t+1$ to repay debt in full, while with probability $1-\alpha$ the borrower will have zero income and will default on the debt. Furthermore, if at time t the borrower incurs a non-pecuniary cost $h[w+b]$, then at time $t+1$ she will be able to divert all repayment funds and pay nothing to the lender. Lenders are competitive, risk-neutral agents, and have an opportunity cost r.

(a) Assuming that the borrower will not divert funds and repay debt whenever she has positive income, at what interest rate would lenders be willing to lend to this borrower?

(b) Using the interest rate derived in (a) above, derive the borrowing constraint that a lender would impose at time t to ensure the borrower does not divert funds and repays debt at time $t+1$ in the state in which she has positive income.

(c) Suppose there is a bailout guarantee that is triggered only if a critical mass of borrowers defaults, in which case the government repays lenders all their promised repayments. What would be the borrowing constraint in this case?

(d) What would be the answers to (c) if we assume instead that a bailout is granted anytime a single borrower defaults?

(e) Use the setup in (a)-(d) to describe an economy that exhibits currency mismatch and lending booms.

(f) Read the paper by Mian and Sufi "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis," QJE 2009. Explain their main finding. Use the setup in (a)-(d) to provide an interpreation of their findings.