Instructions: This exam consists of three parts, and you are to answer all questions. All three parts will receive equal weight in your grade independent of the number of questions contained in that part.

You are allowed six sheets of paper to consult during the exam with notes front and back.

You have four hours to complete this exam.
Part 1

1. Consider a stochastic growth economy with one representative agent and two production sectors. Each sector employs labor and two types of capital: equipment and structures. Denote these by $H$, $K_E$, and $K_S$. Sector 1, the consumption good sector, produces a consumption good, $C$, and structures subject to the following resource constraint:

\[
\begin{align*}
C_t + K_{S,t+1} &= Y_{1,t} + (1 - \delta_s)K_{S,t} = z_t^\theta_1 K_{E,1,t}^\theta_2 K_{S,1,t}^{1-\theta_1-\theta_2} + (1 - \delta_s)K_{S,t}.
\end{align*}
\]

Here, $z_t$ is technology shock where $\log z_{t+1} = \rho_z \log z_t + \epsilon_{z,t+1}$, $\epsilon_t \sim N(0, \sigma_z^2)$.

The second sector, sector 2, uses the same inputs to produce equipment. The resource constraint for this sector is

\[
\begin{align*}
K_{E,t+1} &= Y_{2,t} + (1 - \delta_E)K_{E,t} = q_t z_t^\theta_2 K_{E,2,t}^\theta_1 K_{S,2,t}^{1-\theta_1-\theta_2} + (1 - \delta_E)K_{E,t}.
\end{align*}
\]

In this sector, $q_t z_t$ is the technology shock and $\log q_{t+1} = \rho_q \log q_t + \epsilon_{q,t+1}$, $\epsilon_t \sim N(0, \sigma_q^2)$. Here the function $F$ is homogeneous of degree one. Hence, the productions functions for the two sectors are identical except for the technology shock hitting each sector.

Households have one unit of time to divide between market work and leisure. Preferences are given by $E \sum_{t=0}^{\infty} \beta^t [\log C_t + A \log (1 - H_t)]$.

A. Carefully formulate the dynamic program that would be solved by a social planner in this economy. Be sure to be clear about the state variables and choice variables.

B. Derive expressions that determine how the planner allocates a given amount of capital and labor across the two market sectors. Prove that the same fraction of each input is allocated to a given sector in period $t$. That is, show that $H_{1,t} = \phi_1 H_t$, $K_{E,1,t} = \phi_1 K_{E,t}$ and $K_{S,1,t} = \phi_1 K_{S,t}$. Of course, the remainder, a fraction $(1 - \phi_1)$, is allocated to sector 2.

C. Show that the result obtained in part B can be used to aggregate the resource constraints (*) and (**) into one resource constraint (derive it). Repeat part A given this result. (6 points)

D. Define a recursive competitive equilibrium for this economy. What is the relative price of the output of sector 2? Explain.

E. Consider a certainty version of this economy where $\sigma_1 = \sigma_2 = 0$ but where $z_t$ and $q_t$ grow. In particular, assume $z_t = \gamma^t$ and $q_t = \lambda^t$ where both $\gamma$ and $\lambda$ are greater than one. Characterize the steady state growth path for this economy.
Part 2

In this problem, we consider how the link between growing productivity and wages may be cut off by the endogenous introduction of a new form of capital that substitutes for skilled labor.

We consider a small open economy. We imagine that labor is not mobile across borders. We let $L_S$ and $L_U$ denote the stocks of skilled and unskilled labor in this economy. We imagine that the households and firms in this economy can borrow and lend with people in the rest of the world at a fixed gross interest rate $R$ that does not vary over time. To simplify the model, we assume that neither type of worker (skilled or unskilled) can perform the tasks performed by the other type of worker.

**Households:** Assume that the infinitely-lived representative household in this economy starts off at time 0 with labor endowment $L_S$ and $L_U$ (we imagine that skilled and unskilled workers live together in one big happy household) and owning the representative firm in this economy. We let $\{W_{St}, W_{Ut}\}_{t=0}^{\infty}$ denote the sequence of wage rates and $\{D_t\}_{t=0}^{\infty}$ the sequence of dividends paid to owners of the representative firm. We let $\{C_t\}_{t=0}^{\infty}$ denote the consumption of the representative household.

We assume that this household takes the world interest rate, wage rates, and dividends as given and chooses consumption to maximize utility

$$\sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to a lifetime budget constraint

$$0 = \sum_{t=0}^{\infty} \left( \frac{1}{R} \right)^t [W_{St}L_S + W_{Ut}L_U + D_t - C_t]$$

**Problem A:** 1 point Show that the consumer will choose to have his or her consumption grow at a constant rate

$$\frac{C_{t+1}}{C_t} = \beta R$$

**Problem B:** 1 point If we define $\sum_{t=0}^{\infty} \left( \frac{1}{R} \right)^t [W_{St}L_S + W_{Ut}L_U + D_t]$ as the lifetime wealth of this consumer, what fraction of that wealth does the consumer consume at time zero ($C_0$)? Does that depend on the interest rate? (Note that lifetime wealth does depend on the interest rate, but the question is whether the fraction of that lifetime wealth that is consumed depends on the interest rate.)
**Firms:** Assume that the representative firm in this economy owns the initial capital stocks $K_{10}, K_{20}$ and chooses to hire labor $\{L_{St}, L_{Ut}\}_{t=0}^{\infty}$ and investment and future capital stocks $\{X_{1t}, X_{2t}, K_{1t+1}, K_{2t+1}\}_{t=0}^{\infty}$ as well as dividends $\{D_t\}_{t=0}^{\infty}$ to maximize the discounted present value of dividends

$$\sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t D_t$$

subject to the constraints that dividends are given by

$$D_t = Z_t K_{1t}^\alpha (K_{2t} + L_{St})^\gamma L_{Ut}^\nu - W_{St} L_{St} - W_{Ut} L_{Ut} - X_{1t} - X_{2t}$$

and that capital stocks evolve according to

$$K_{1t+1} = (1 - \delta_1) K_{1t} + X_{1t}$$

$$K_{2t+1} = (1 - \delta_2) K_{2t} + X_{2t}$$

and that $K_{1t+1} \geq 0$ and $K_{2t+1} \geq 0$.

Assume that $Z_t$ grows at a constant rate

$$\frac{Z_{t+1}}{Z_t} = \exp(g_Z)$$

Define an equilibrium in this economy as an allocation $\{C_t, L_{St}, L_{Ut}, X_{1t}, X_{2t}, K_{1t+1}, K_{2t+1}, D_t\}_{t=0}^{\infty}$ together with wages $\{W_{St}, W_{Ut}\}_{t=0}^{\infty}$ and the world interest rate $R$ such that the allocation of consumption maximizes household utility subject to the household budget constraint taking wages, interest rates, and dividends as given and such that the sequence of labor, investment, capital stocks, and dividends, maximizes the discounted present value of dividends subject to the firm’s constraints and such that labor markets clear in the sense that $L_{St} = L_S$ and $L_{Ut} = L_U$.

**Problem C: 1 point** We can define the trade balance of the country as output less consumption and investment or

$$TB_t = Y_t - C_t - X_{1t} - X_{2t}$$

with

$$Y_t = Z_t K_{1t}^\alpha (K_{2t} + L_{St})^\gamma L_{Ut}^\nu$$
Can you prove that in equilibrium, the discounted present value of the country’s trade balances are zero?

**Hint:** Use the definition of dividends and the budget constraint of the representative household.

**Problem D: 2 points** Set up the firm’s problem as a Lagrangian and derive the following first order conditions

\[
W_{St} = \gamma \frac{Y_t}{K_{2t} + L_{St}}
\]

\[
W_{Ut} = \nu \frac{Y_t}{L_{Ut}}
\]

\[
R \geq \alpha \frac{Y_t}{K_{1t+1}} + (1 - \delta_1)
\]

\[
R \geq \gamma \frac{Y_t}{K_{2t+1} + L_{St+1}} + (1 - \delta_2)
\]

with

\[
Y_t \equiv Z_t K_{1t}^\alpha (K_{2t} + L_{St})^\gamma L_{Ut}^\nu
\]

with these last two first order conditions being equalities when \(K_{1t+1} > 0\) and \(K_{2t+1} > 0\) respectively.

**Problem E: 2 points** Describe how to solve for \(W_{St+1}\) and \(W_{Ut+1}\) and \(K_{2t+1}\) as a function of \(Z_{t+1}\), the labor stocks \(L_S, L_U\) and the parameters of the model.

**Problem F: 2 points** Show that there is a cutoff level of productivity \(\bar{Z}\) such that if \(Z_{t+1} \leq \bar{Z}\), then \(K_{2t+1} = 0\) while if \(Z_{t+1} > \bar{Z}\), then \(K_{2t+1} > 0\).

**Problem G: 1 point** Show that if \(Z_{t+1} > \bar{Z}\), then the wage for skilled labor is constant at \(W_{St+1} = R - (1 - \delta_2)\) no matter how large productivity gets.
Part 3  Question 1: Trade and structural transformation

Structural transformation is the process by which economic growth coincides with changes in the distribution of economic activity across broad sectors of an economy — agriculture, industry, and services. In this question we focus on reallocation of labor between the manufacturing sector (internationally tradable) and services (non-tradable).

Production of the final good, used for consumption, in country $i \in S$ is:

$$C_i = \left[ (C_{Mi})^{\frac{\rho-1}{\sigma}} + (C_{Si})^{\frac{\rho-1}{\sigma}} \right]^{\frac{\sigma}{\rho-1}},$$

(1)

where $C_{Mi}$ and $C_{Si}$ denote consumption of manufactured goods and services in country $i$. The parameter $\rho \neq 1$ is the elasticity of substitution in final good between manufacturing and services.

Consumption of manufactured goods in country $j$, $C_{Mj}$, is an Armington aggregator over manufactured goods produced in different source countries:

$$C_{Mj} = \left( \sum_{i \in S} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Manufactured goods and services are produced in each country using labor according to

$$Q_{Mi} = A_{Mi}L_{Mi},$$

$$Q_{Si} = A_{Si}L_{Si}.$$

Labor can be freely reallocated across sectors, satisfying the resource constraint

$$L_{Mi} + L_{Si} = L_i,$$

where $L_i$ is the exogenous labor endowment in country $i$.

Manufacturing output in each country must equal its use across all countries:

$$Q_{Mi} = \sum_j \tau_{ij}q_{ij}.$$

For non-traded services,

$$Q_{Si} = C_{Si},$$

where $Q_{Si}$ is output of services in country $i$.

All markets are competitive. Labor earns the value of its marginal product in each sector and producer prices equal marginal cost in each sector.

Finally, assume that trade is balanced in each country. That is, in each country $i \in S$,

$$p_{Mi} \sum_{j \neq i} \tau_{ij}q_{ij} = \sum_{j \neq i} p_{Mj}\tau_{ji}q_{ji},$$

where $p_{Mi}$ is the producer manufacturing price in country $i$.

1. Suppose that country $i$ is in autarky (i.e. $\tau_{ij} = \tau_{ji} = \infty$ for $j \neq i$). How does an increase in manufacturing productivity $A_{Mi}$ shift employment between manufacturing and services? Under what conditions does employment shift from manufacturing to services? Provide intuition for your answer.

For the next questions, consider an equilibrium with international trade.
2. Provide an expression for the “consumption” price of manufactured goods in country $i$, $P_{Mi}$, in terms of worldwide producer prices $p_{Mj}$ and trade costs $\tau_{ji}$ for $j \neq i$.

3. Show that, with balanced trade,

$$P_{Mi}C_{Mi} = w_iL_{Mi}.$$  

4. Provide an expression for the ratio of producer manufacturing prices to consumer manufacturing prices in country $i$, $\frac{p_{Mi}}{P_{Mi}}$, in terms of country $i$’s own-trade share

$$\lambda_{ii} = \frac{p_{Mi}q_{ii}}{P_{Mi}C_{Mi}},$$

and other country $i$’s parameters.

5. Write an expression for the allocation of labor between manufacturing and services in country $i$ in terms of “sufficient statistics” in country $i$ (including own trade-share).

6. Consider a reduction in trade costs that increases country $i$’s international trade share. How does this shock shift employment between manufacturing and services? Under what conditions does this reduction in trade costs shift employment from manufacturing to services? Provide intuition for your answer.
Part 3 Question 2 Price Setting with Sticky Prices and Variable Markups

Consider firm i profit maximization

$$\max_{P_i} \Pi_{it} = (P_{it} - W_t/A_{it})C_{it}$$

with the following demand schedule:

$$C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta_t} C_t,$$

where the firm takes \((W_t, A_{it}, C_t, P_t)\) and \(\eta_t\) as given. Assume for simplicity symmetric firms with \(A_{it} = A_i\) for all \(i\).

1. Explain all terms in the problem of the firm. What can be a simple microfoundation for time-varying \(\eta_t\)? Show that the optimal static markup is \(\eta_t / (\eta_t - 1)\), that is the profit maximizing price is \(\bar{P}_{it} = \frac{\eta_t W_t}{\eta_t - 1 A_t}\).

2. Set up the price setting of a firm that is subject to a Calvo adjustment friction, where it can adjust prices with probability \(1 - \theta\) each period. Characterize the optimal reset price \(\bar{P}_{it}\) as a forward looking weighted average of future markups and marginal costs.

3. Define the desired price in logs as:

$$\tilde{p}_{it} = \mu_t + mc_t,$$

where \(\mu_t \equiv \log \frac{\eta_t}{\eta_t - 1}\), \(mc_t \equiv w_t - a_t\).

Explain the meaning of the desired price and each term. Show that the log-linearized Calvo reset price is given by:

$$\bar{p}_{it} = (1 - \beta \theta)^\infty \sum_{j=0}^\infty (\beta \theta)^j E_t \{\mu_{t+j} + mc_{t+j}\},$$

where \(\beta\) is the discount factor.

4. Assume that \(\mu_t = -\alpha(p_{it} - p_t)\), where \(p_{it} - p_t \equiv \log(P_{it}/P_t)\). Can you think of a microfoundation for such a model and why parameter \(\alpha \geq 0\) is sometimes called the elasticity of strategic complementarity (in pricing). Argue [without derivation, if short on time] that the resulting Phillips curve is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(w_t - p_t - a_t),$$

where \(\lambda = \frac{(1 - \alpha)(1 - \theta)(1 - \beta \theta)}{\theta}\).

Interpret the Phillips curve and in particular the term \(\lambda(w_t - p_t - a_t)\). Why is the slope of the Phillips curve flatter when \(\alpha\) is greater?