# UCLA

# Department of Economics

# Ph. D. Preliminary Exam

# Microeconomic Theory

# Spring 2023

Instructions:

- You have 4 hours for the exam
- Answer all the questions
- Each section is weighted equally
- You are allowerd 6 double-sided sheets (12 sides) of notes and a non-graphical calculator

## 1) A Magical Brownie Economy

Jack and Jill are the only two consumers in an economy with two goods: chocolate ($x$) and brownies ($y$). Both have initial endowments $c > 0$ of chocolate and no brownies. Brownies are "magical" in that their total amount can double with probability $r \in (0, 1)$ (this occurs after purchase and at the moment of consumption). Jack and Jill are expected utility maximizers with vNM utility

$$u_i(x, y) = 1 - e^{-x-y}$$

Both own equal shares in two bakeries that make brownies with the production function

$$f_j(x) = \log(x + 1)$$

(a) Calculate the demand for each consumer.

(b) Calculate the supply for each bakery.

(c) Solve for a competitive equilibrium.

(d) How is the price of brownies affected by their "magical quality", i.e. $r$. Can one infer $r$ from observed prices in this economy?

(e) How are prices affected by the initial endowment of chocolate? Provide some intuition.

## 2) Commitment and Conflict

UCLA's Micro and Macro groups are fighting over next year's hiring budget, which we normalize to 1. Assume first that both submit a demand (or "commitment") $s_i \in [0, 1]$. If the demands are compatible, $s_1 + s_2 \leq 1$, then both get their demand and split the remainder equally, for utility $u_i = s_i + \frac{1-(s_1+s_2)}{2}$. If the demands are incompatible, $s_1 + s_2 > 1$, no one is hired (and the dean sends everybody to a team-building exercise), for utility $u_i = 0$.

(a) Characterize the pure strategy Nash equilibria.

From now on assume that committing has cost $c \in (0, 1/2)$, but players don't have to commit, and can instead wait; strategy sets are thus $S_i = [0, 1] \cup \{w\}$. If both players wait, they split the budget equally for payoffs $u_i = 1/2$. If one waits $s_i = w$ and the other commits $s_{-i} \in [0, 1]$, the committed party gets its demand, for utility $u_{-i} = s_{-i} - c$, while the other takes the residual $u_i = 1 - s_{-i}$. If both players commit, payoffs are as in part (a), minus $c$ for each player.

(b) Which of the equilibria from part (a) are equilibria in this new game, where players can also wait?

(c) Show that committing to $s_i \in [0, 1/2]$ is strictly dominated![1]

(d) Show that committing to $s_i \in (1/2, 1)$ is iteratively strictly dominated! (Hint: Recall that a strategy may be dominated by a mixed strategy.)

(e) Solve for the pure and mixed strategy equilibria of this game.

(f) Now assume that the commitment cost is $c = 0$. Which of the equilibria from part (a) are equilibria now? Which of these equilibria use weakly dominated strategies? Characterize the pure strategy equilibria in weakly undominated strategies in this game.

## 3) Auctions with Entry

A principal has a single object to sell via an auction. There is a large number of potential bidders; each can pay entry cost $k$ to enter the auction (e.g. the cost of travelling to the auction).[2] After entering, bidders learn their IID private value $\theta_i \sim F[\underline{\theta}, \overline{\theta}]$. A direct mechanism is described by $\langle n, p_i, t_i \rangle$. The principal first chooses $n$ bidders to enter (assume $n$ is deterministic). Each entering bidder $i$ learns his type $\theta_i$ and reports $\tilde{\theta}_i$, which determines his probability of winning $p_i(\tilde{\theta})$ and his payment $t_i(\tilde{\theta})$, where $p_i \in [0, 1]$ and $\sum_i p_i \leq 1$. As usual, $\tilde{\theta} := (\tilde{\theta}_1, \ldots, \tilde{\theta}_n)$ is the vector of everyone's reports and $\tilde{\theta}_{-i}$ is everyone's reports except $i$. If agent $i$ enters and reports $\tilde{\theta}_i$, while other agents truthfully report their types, $i$'s interim expected utility is

$$u_i(\theta_i, \tilde{\theta}_i) = E_{-i}\left[\theta_i p_i(\tilde{\theta}_i, \theta_{-i}) - t_i(\tilde{\theta}_i, \theta_{-i})\right] - k.$$

An agent who does not enter obtains zero utility. The principal's ex-ante expected profit is

$$\Pi = E\left[\sum_i t_i(\theta) + \left(1 - \sum_i p_i(\theta)\right)\theta_0\right],$$

where the principal has known valuation $\theta_0$.

We first derive the principal's profit given $n$ agents enter.

(a) Show that incentive compatibility (IC) implies that interim utility obeys an integral equation and a monotonicity constraint.

(b) Write down the ex–ante individual rationality (IR) constraint that ensures each entering bidder is willing to pay the entry cost (recall the agent does not know his type when choosing whether to enter).

(c) Write down the principal's problem of maximizing her profit subject to (IC) and (IR).

[1] Note that you can solve part (e) taking the conclusions of parts (c) and (d) as given, rather than solving them.
[2] The "large number" of potential bidders means we will never run out of entrants.

(d) What is the principal's profit-maximizing mechanism? Show her profit-maximizing allocation is efficient.

(e) Intuitively, why does the solution in (d) differ from the optimal profit-maximizing auction from class whereby the principal allocates the object to the agent with the highest $MR(\theta_i)$, so long as $MR(\theta_i) \geq \theta_0$? (Assuming $MR(\theta_i) := \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}$ is increasing).

We now turn to the optimal entry decision and to implementation.

(f) Let $\Pi(n)$ be the profit from $n$ bidders. Show $\Pi(n)$ is strictly concave and thus obtains a maximum $n^*$ characterized by $\Pi(n^*) - \Pi(n^* - 1) \geq 0 \geq \Pi(n^* + 1) - \Pi(n^*)$.

(g) Argue that the profit-maximizing allocation can be implemented by a second-price auction (SPA) with reserve price. What is the reserve price?

(h) Suppose we run the SPA in (g) and bidders make their entry decisions sequentially. Argue that bidder $n$'s expected utility when he enters coincides with $\Pi(n) - \Pi(n-1)$ and thus, if $\Pi(n^*) - \Pi(n^* - 1) = 0$, profit is maximized by free entry.[3]

[3] Aside: If $\Pi(n^*) - \Pi(n^* - 1) > 0$ then the last entrant makes positive utility and the profit-maximizing mechanism has a small entry fee to eliminate agents' rents.