UCLA

## Department of Economics

Ph. D. Preliminary Exam

## Microeconomic Theory

## Fall 2023

Instructions:

- You have 4 hours for the exam
- Answer all the questions
- Each section is weighted equally
- You are allowerd 6 double-sided sheets (12 sides) of notes and a non-graphical calculator


## 1) A Simple Economy

Consider an economy with 2 consumers, 2 firms and 2 goods. Both consumers have the same endowment $e>0$ of good 1 and zero of good 2 . They have the same utility

$$
u_{i}\left(x_{1}, x_{2}\right)=\left(\sqrt{x_{1}}+\sqrt{x_{2}}\right)^{2}
$$

Consumers own equal shares of both firms that have the same production function of $x_{2}$ :

$$
f_{j}\left(x_{1}\right)=\sqrt{x_{1}}
$$

(a) Calculate aggregate demand and supply.
(b) Solve for an equilibrium.
(c) Show that the price of good 1 is decreasing in $e$.

## 2) Reputational Cheap Talk

The President is deciding between two alternatives: enact a new educational program (E) or not enact and stay with the status quo ( N ). The status quo will give the president a (commonly known) payoff of 0 . The payoff from the new education program depends on the state of the world. With probability $1 / 4$ the new program will be good (G) and give the President a payoff of 1 . With probability $3 / 4$, it will be bad (B) and provide a payoff of -1 . Hoping to obtain sage advice, the President hires an Economist. He is uncertain, however, of the Economist's competence. The Economist is knowledgeable with probability $1 / 2$, and ignorant with probability $1 / 2$ (independently of the state of the world). If the Economist is knowledgeable (K), he is aware of the state of the world. If the Economist is ignorant (I), he does not know the state of the world (and therefore believes that the program will be good with probability 1/4). The Economist knows his own type. The game proceeds as follows:

- Stage 1: Nature chooses the state of the world (G or B) and the type of Economist (K or I). The knowledgeable Economist observes the state of the world.
- Stage 2: The Economist advises the President about the state of the world. Specifically, he sends one of two messages, either g (for good) or b (for bad). The Economist cannot profess ignorance, and is constrained to send one of those two messages.
- Stage 3: The President, viewing the advice, chooses either E or N.

After viewing his payoff (either $-1,0$, or +1 ), the President, who is a good Bayesian, infers that the Economist is knowledgable with probability $\mu$. The Economist's payoff is simply $\mu$ (he cares about
his future reputation). All players are risk-neutral. Throughout, apply the (weak) PBE equilibrium concept.
(a) Describe each player's strategy set.
(b) Now you will explore the possibility that there is an equilibrium in which the knowledgeable Economist gives informative advice based on the state of the world ( $g$ when $G$ and $b$ when $B$ ), and the ignorant Economist always sends the message b.
(i) In such an equilibrium, what would be the probability that the President receives the message g ? What would be his belief about the state of the world given he receives g ? What will the President do when he receives the message g?
(ii) In such an equilibrium, what would be the probability that the President receives the message b ? What is his belief about the state of the world given he receives the message b? What will the President do when he receives the message b?
(iii) In such an equilibrium, what would be the President's Bayesian beliefs about the probability that the Economist is knowledgeable given the combinations of message and payoff ( $\mathrm{g}, 1$ ) and $(\mathrm{b}, 0)$ ? What is range of beliefs after $(\mathrm{g},-1)$ ?
(iv) Is there an equilibrium of the form we have been examining? If not, explain why not. (Remember that the President's final beliefs about the Economist's type determine the Economist's payoff.)
(c) How do your answers to part (b) change if the probability of the Economist being knowledgeable is $1 / 4$ ? Interpret your results.

## 3) Selling Concessions

A stadium (the "principal") is selling a concession to an agent to sell hot dogs at a baseball game. The agent has private information about consumers' demand for hot dogs, $\theta \sim F[\underline{\theta}, \bar{\theta}]$. Demand is linear with $p=\theta-q$, where $q$ is the quanity sold. The agent's cost is zero, so his utility is

$$
u=q(\theta-q)-t
$$

where $t$ is the transfer paid to the principal. The principal's profit is $\pi=t$. Assume $\frac{1-F(\theta)}{f(\theta)}$ is decreasing in $\theta$.

Consider the first-best problem. Suppose that the principal can observe the agent's type $\theta$ and fully extract his rents.
(a) What is the first-best allocation $q(\theta)$ ?
(b) Suppose $\theta \sim U[0,1]$. Calculate the principal's expected profit.

For the rest of the question, suppose $\theta$ is only known by the agent. Suppose that the quantity of hot dogs sold $q$ is contractible. A mechanism $\langle q(\tilde{\theta}), t(\tilde{\theta})\rangle$ consists of quantity $q(\tilde{\theta}) \in \mathbb{R}_{+}$and transfer, $t(\tilde{\theta}) \in \mathbb{R}$, where $\tilde{\theta}$ is the agent's reports type.
(c) Write down the agent's expected utility in an IC mechanism.
(d) What is the principal's profit maximizing mechanism?
(e) Suppose $\theta \sim U[0,1]$. Calculate the principal's expected profit. Show that it is less than the profit in (b).

Now, suppose that the quantity $q$ is not contractible, so once the principal sells the concesession to the agent, the agent can sell however much they like. A mechanism $\langle y(\tilde{\theta}), t(\tilde{\theta})\rangle$ consists of an allocation probability $y(\tilde{\theta}) \in[0,1]$ and transfer $t(\tilde{\theta}) \in \mathbb{R}$.
(f) Fix $\langle y, t\rangle$. How will the agent choose the quantity sold, $q$, if they buy the concession?
(g) What is the principal's profit maximizing mechanism?
(h) Suppose $\theta \sim U[0,1]$. Calculate the principal's expected profit. Show that it is less than the profit in (e).

