UCLA Department of Economics **Fall 2023**

PhD Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts. There is one question in Part 1, one question in Part 2, and two questions in Part 3. Each part counts for 1/3 of your overall score on this exam, independent of the number of questions contained in that part. The allocation of points indicated within each part of this exam is simply a guide to the relative importance of each part of a multipart question. Your final scores on each part of this exam will be normalized so that each of the three parts gets equal weight.

You have four hours to complete this exam.

Suggestions for time management: You should allocate adequate time at the start of this exam to read this all the questions and decide which questions you wish to answer first. You should be sure to be mindful of the time if you get stuck on some part of the exam. If you are stuck, move on to other questions that you feel you can answer.

Part 1. The Economic Impact of the Louisiana Purchase

The *Louisiana Purchase* (1803) roughly doubled the land size of the United States. Develop a general equilibrium model to assess the economic impact of the Louisiana Purchase as described below.

There is no specific correct answer; rather, you will be graded individually on how well you apply dynamic general equilibrium analysis to address the question.

Please write clearly and darkly. **Unreadable answers will receive no credit**.

Read the entire question before answering any part of it.

(i) There are two consumption goods, food and non-food consumption. Food accounts for about 50 percent of GNP, investment about 20 percent of GNP, non-food consumption accounts for about 30 percent of GNP. Investment and non-food consumption are produced from the same technology.

(ii) There are three production inputs for both goods: labor, capital and land. Land's share of income in food production is 20 percent, and is 5 percent in non-food production.

(iii) Before the Louisiana Purchase, there is one unit of land. After the Louisiana Purchase, there are 2 units of land. The land acquired in the Louisiana Purchase has a comparative advantage in producing food.

The cost of the Louisiana Purchase was $18 per square mile, in 1803 dollars. The purchase was equal to 3 percent of GNP in 1803. Today, the average value of that land per square mile is about $3 million, in 2023 dollars. The price level today is about 17 times higher than it was in 1803.

(A) Summarize in words only, the key economic issues involved, and your ideas of how you will address these issues.

(B) Write down your **pre-Louisiana Purchase model**, including all your assumptions about preferences, endowments, technologies, constraints.

(C) Write down the equations and constraints and any other information that characterize the model's solution.

(D) Show the equations that characterize the steady state of the model. This should include a price equation for land.

(E) Modify the model to **incorporate the Louisiana Purchase**, including your assumptions about endowments, technologies, constraints.

(F) Write down the equations and constraints that characterize the model's solution. This should include price equations for the two types of land.

(G) - Show the equations that characterize the steady state of the model

(H) Describe an algorithm that could be used to construct the transition path between the two steady states. What do you predict about the growth rate of the economy after the Louisiana Purchase? Explain your answer.

(I) Use the model to show how to evaluate the benefits versus the cost of the Louisiana purchase.

(J) Based on your analysis and the information provided, do you think that the purchase price was a fair market price? Explain your answer.

Part 2

Points for parts of this problem add up to 10 points.

In this problem we consider the implications of two models for asset prices and the dynamics of inequality in the population. The models are identical except in one key dimension. In the first model, all individuals face the same stochastic shocks to their endowments, so that the aggregate and individual's endowment are the same. In the second model, the shocks are independent across individuals so that the aggregate endowment is deterministic.

Both models are in discrete time $t \in \{0, 1, 2, \ldots\}$. The economy is populated with a measure one continuum of individuals who start with the same endowment of the single final consumption good denoted by y_0. At the start of every period, each individual experiences an endowment growth shock g_t so that individuals' endowments in period $t \geq 1$ are given by $y_t(h^t) = g_t y_{t-1}(h^{t-1})$, where $h^t = (y_0, g_1, g_2, \ldots, g_t)$ is the time-t history of initial endowment and endowment growth shocks, and $y_t(h^t)$ is the endowment at time $t \geq 1$ of an individual. In both models, the growth shocks are independently and identically distributed over time and can take one of two values $0 < g_L < g_H$. The probability that $g_t = g_H$ is given by $\pi \in (0, 1)$ and the probability that $g_t = g_L$ is given by $1 - \pi$. Finally, we assume that agents have time and state separable logarithmic utility over consumption streams:

$$\sum_{t=0}^{\infty} \sum_{h^t} \beta^t \pi_t(h^t) \log(c_t(h^t)),$$

where $\pi_t(h^t)$ is the probability as of time $t = 0$ that history h^t is realized in period $t \geq 1$ for a particular individual.

The only difference between the two models is the following. In the first model, the endowment growth shocks are aggregate, in the sense that all individuals experience the same history h^t of growth rate shocks up through time t for all $t \geq 1$. Thus, aggregate and individual endowments are the same:

$$Y_t(h^t) = y_t(h^t), \ \underline{\text{in the first model}},$$

where $Y_t(h^t)$ is the aggregate endowment at time $t \geq 1$. In the second model, the individuals each experience idiosyncratic endowment growth shocks, implying that the aggregate endowment at time t is deterministic and given by

$$Y_t = \sum_{h^t} \pi_t(h^t) y_t(h^t), \ \underline{\text{in the second model}}.$$

1. (1pt) For a given individual, how many different histories are possible in period 4? What is $\pi_4(y_0, g_L, g_L, g_H, g_H)$ in terms of the parameter π? What is $y_4(y_0, g_L, g_L, g_H, g_H)$?

2. (1pt) This question studies individual and aggregate endowment growth in the two models.

   (a) (0.5pt) As a function of the parameters π, g_L, g_H, calculate

   $$\mathbb{E}\left[\log\left(y_{t+1}(h^{t+1})\right) - \log\left(y_t(h^t)\right) \mid h^t\right]$$
   $$\mathbb{V}\left[\log\left(y_{t+1}(h^{t+1})\right) - \log\left(y_t(h^t)\right) \mid h^t\right],$$

   the expectation and the variance of the growth rate of a individual's endowment from t to $t+1$. Explain why this expectation and variance are the <u>same</u> in both models.

   (b) (0.5pt) As a function of the parameters π, g_L, g_H, calculate

   $$\mathbb{E}\left[\log\left(Y_{t+1}(h^{t+1})\right) - \log\left(Y_t(h^t)\right) \mid h^t\right]$$
   $$\mathbb{V}\left[\log\left(Y_{t+1}(h^{t+1})\right) - \log\left(Y_t(h^t)\right) \mid h^t\right],$$

   the expectation and the variance of the growth rate of the <u>aggregate</u> endowment from t to $t+1$. Explain why this expectation and variance are <u>different</u> in both models.

3. (1pt) Feasibility in the two models.

   (a) (0.5pt) <u>In the first model</u>, what condition must an allocation $\{c_t(h^t)\}_{t=0}^{\infty}$ satisfy to be feasible.

   (b) (0.5pt)<u>In the second model</u>, what condition must an allocation $\{c_t(h^t)\}_{t=0}^{\infty}$ satisfy to be feasible.

4. (3pt) <u>In the first model</u>, individuals have access to trading shares of the aggregate endowment and a one-period risk free bond. We assume that at the start of time $t = 0$, individuals have neither shares nor the risk free bond. Let $\{s_t(h^t)\}$ denote the shares of the aggregate endowment and let $\{b_t(h^t)\}$ denote the corresponding quantity of risk free bonds that an individual purchases in period t following history h^t. For simplicity, we assume that individuals face borrowing and short-selling constraint that

never bind in equilibrium. Let $\{p_t(h^t)\}$ denote the prices of shares of the aggregate endowment and $\{q_t(h^t)\}$ the prices of one-period risk free bonds. We can write the sequence of budget constraints for an individual in this first model for $t \geq 1$ as

$$c_t(h^t) + q_t(h^t)b_t(h^t) + p_t(h^t)s_t(h^t) = y_t(h^t) + b_{t-1}(h^{t-1}) + (y_t(h^t) + p_t(h^t))s_{t-1}(h^{t-1}),$$

where $s_{-1} = b_{-1} = 0$, and where we do not explicitly state the borrowing and short-selling constraints since they do not bind.

(a) (1pt) State the individual optimization problem and derive its first order conditions (Euler equations).

(b) (1pt) Keeping in mind that the aggregate endowment of risk-free bond and shares of the aggregate endowment is zero, define an equilibrium.

(c) (0.5pt) In this first model, one can show that there is an equilibrium in which $c_t(h^t) = y_t(h^t) = Y_t(h^t)$ at all dates and after all histories, in which asset holdings $s_t(h^t) = b_t(h^t) = 0$ at all dates and states, and in which the bond prices $q_t(h^t)$ are all equal to a constant q and the share prices $p_t(h^t)$ are all equal to a constant times aggregate output $py_t(h^t)$. For this equilibirum, solve for q and p from your Euler equations in the previous question as functions of the parameters β, π, g_L, g_H.

(d) (0.5pt) Denote the return on the risk free bond by $R^f_{t+1} = 1/q_t(h^t) = 1/q$. Denote the realized return on a share of the aggregate endowment by

$$R^s_{t+1}(h^{t+1}) = \frac{y_{t+1}(h^{t+1}) + p_{t+1}(h^{t+1})}{p_t(h^t)} = \frac{y_{t+1}(h^{t+1})}{y_t(h^t)} \frac{1+p}{p}.$$

Compute the equity risk premium given by

$$\mathbb{E}_t R^s_{t+1} - R^f_{t+1}$$

as a function of the parameters β, π, g_L, g_H.

5. (3pt) Consider now the same asset markets but in <u>the second model</u>. Since there is no aggregate uncertainty, the price of assets is now deterministic. We denote by p_t the price of a share to the aggregate endowment at time t, and by q_t the price of a bond at time t.

(a) (0.5pt) What are the differences between the individual optimization problem in the first and the second model? Using your answer together with the work you have already done for question 4(a), state the first order conditions (Euler equations) of the individual optimization problem in the second model.

(b) (0.5pt) Keeping in mind that the aggregate endowment of risk-free bond and shares of the aggregate endowment is zero, define an equilibrium.

(c) (1pt) In this second model, one can show that there is an equilibrium in which $c_t(h^t) = y_t(h^t)$ at all dates and after all individual histories, in which asset holdings $b_t(h^t) = s_t(h^t) = 0$ at all dates and states, and in which the bond prices $q_t = q$ are all equal to a constant q, and the stock price is equal to $p_t = pY_t$. Show that this constant q is the same as you derived in question 5(c). Using the Euler equation, derive a recursive equation for p.

(d) (1pt) Show that the equity premium is equal to zero and explain why.

6. (1pt) In the equilibrium with $c_t(h^t) = y_t(h^t)$ in the first model, how does the cross section (across individuals) dispersion of log consumption evolve over time? It it constant? Does it grow over time? Explain why. In the equilibrium with $c_t(h^t) = y_t(h^t)$ in the section model, how does the cross section (across individuals) dispersion of log consumption evolve over time? It it constant? Does it grow over time? Explain why.

Part 3 **[Split your time equally between the two questions.]**

3.A International trade and the skill premium

In this question, we consider a specification of the Armington model in which each country produces output using two labor types (high and low skilled labor) and intermediate inputs (materials). Intermediate inputs are made of the same final good as consumption. That it, intermediate inputs contain the same import content as consumption.

Output in country i is produced according to

$$Q_i = F_i\left(H_i, L_i, M_i\right) = \left[\left(A_{Hi} H_i^{\alpha} M_i^{1-\alpha}\right)^{\frac{\rho-1}{\rho}} + \left(A_{Li} L_i\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

where H_i and L_i denote high- and low-skilled labor (in fixed supply) and M_i denotes the use of intermediate input. The parameter $\rho \neq 1$ is the the elasticity of substitution between low-skilled labor and the skilled-labor / materials composite.

The intermediate input is made of the same final good that is used for consumption. Specifically, we assume that a final good is produced in each country j according to the Armington aggregator, $\left(\sum_{i\in S} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, with $\sigma > 1$. Given this technology, competitive final good firms purchase individual goods q_{ij} to produce the final good. The final good is then used for production (M_j) and for consumption by households (C_j). Households derive utility from consumption of the final good, $u\left(C_j\right)$. The resource constraint for the final good in country j is given by

$$\left(\sum_{i\in S} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = M_j + C_j.$$

The resource constraint for output produced in country i is

$$Q_i = \sum_j \tau_{ij} q_{ij}.$$

All markets are competitive. Each labor group earns the value of its marginal product:

$$w_i = p_{ii}\frac{\partial F_i}{\partial L_i} \qquad \text{and} \qquad s_i = p_{ii}\frac{\partial F_i}{\partial H_i},$$

where p_{ii} is the output price in country i. We denote the price of the final good in country i by P_i.

1. (0.2 points) Write an expression for the skill premium in country i, s_i/w_i, in terms of L_i, H_i, M_i and the productivity parameters.

2. (0.2 points) Suppose that there is an exogenous increase in the quantity of intermediate inputs used in country i, M_i. Provide a condition on parameters such that the skill premium

in country i rises. Provide intuition for your answer.

In the following questions, we endogenize the change in M_i in response to a move to autarky. Note that the quantity M_i must satisfy the first order condition

$$P_i = p_{ii} \frac{\partial F_i}{\partial M_i}.$$

3. (0.3 points) Write an expression for the relative price p_{ii}/P_i in terms of the domestic share of gross output, λ_{ii},

$$\lambda_{ii} \equiv \frac{p_{ii} q_{ii}}{P_i \left(C_i + M_i\right)},$$

and other model parameters.

4. (0.3 points) Suppose that, starting in a trade equilibrium in which $\lambda_{ii} < 1$, country i moves to autarky in which $\tau_{ij} = \infty$ for $j \neq i$. All other parameters remain unchanged. What is the impact of this move to autarky on country i's skill premium? You do not need to fully characterize the solution analytically, but you need to show what equations you use to obtrain your answer.

3.B Inflation dynamics

Consider the following log-linear model of price setting and price level dynamics:

$$\begin{aligned}
\bar{p}_t &= (1-\beta\theta) \sum\nolimits_{j=0}^{\infty} (\beta\theta)^j E_t \tilde{p}_{t+j}, \\
\tilde{p}_{t+j} &= \alpha p_t + (1-\alpha) m_t, \\
p_t &= \theta p_{t-1} + (1-\theta)\bar{p}_t, \\
\Delta m_t &= \rho \Delta m_{t-1} + \varepsilon_t.
\end{aligned}$$

(i) Explain each equation. What is the role of θ and α? Why is m_t a measure of aggregate demand?

(ii) Derive the Phillips curve, $\pi_t = \beta E_t \pi_{t+1} + \lambda(m_t - p_t)$, where $\pi_t = \Delta p_t$. What is the value of λ and how does it depend on θ and α, and why? Why is $(m_t - p_t)$ a measure of output gap?

(iii) For $\alpha = \rho = 0$, solve for the dynamics of inflation π_t and reset-price inflation $\bar{p}_t = \Delta \bar{p}_t$. What processes do these two series follow? If there is a one-time permanent expansion in aggregate demand m_t, could this model account for persistent inflation? persistent reset-price inflation?

(iv) Redo part (iii) for $\rho > 0$. How do your answers change? What if instead of $\rho > 0$, there is $\alpha > 0$? What are the likely source of persistent reset-price inflation?