UCLA Department of Economics **Fall 2022**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to answer all questions. All three parts will receive equal weight in your grade independent of the number of questions contained in that part.

You have four hours to complete this exam.

Part I

1. Consider a stochastic growth model where a representative household's preferences are given by,

$$E\sum_{t=0}^{\infty}\beta^t\left\{\log c_t - A\frac{h_t^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right\},\ A>0,\ \omega>0,$$

where $0<\beta<1$. For each $t\geq 0$, the technology is given by,

$$c_t + i_t = e^{z_t}\left(u_t k_t\right)^{\theta} h_t^{1-\theta}\ \text{, where}$$

$$k_{t+1} = (1-\delta_t)k_t + i_t\ ,$$

$$\delta_t = \delta u_t^{\phi}\ \text{and}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}\ .$$

As usual, ε is an i.i.d. random variable with mean 0 and variance σ_ε^2. The variable u is a choice variable describing the intensity with which the capital stock is utilized in a given period, where $0\leq u_t\leq 1$. The parameter $\phi>1$ and $0<\delta<1$. The idea is that when capital is used more intensely, it depreciates faster.

A. Carefully state the representative agent's dynamic programming problem for this economy.

B. Define a *recursive competitive equilibrium* for this economy. Hint: Consider a market for utilized capital services rather than a capital rental market.

C. The *Frisch elasticity of labor supply* is defined to be the elasticity of hours worked to the wage rate holding the marginal utility of wealth (the Lagrange multiplier for the budget constraint) constant. Derive the Frisch elasticity for your decentralized economy. Make sure you clearly explain your derivation.

Question 2 of Part I

Housing In General Equilibrium

There is a representative household that has preferences defined over sequences of non-durable consumption (c) and housing (h). The economy is perfectly competitive and is perfect foresight. Preferences, technologies, constraints are below. Capital letters are aggregate variables. Lower case letters are per-capita variables. The size of the population is normalized to one. The initial capital, K_0, is given, as is the initial stock of housing H_0.

Read through the entire question before answering. *Hint: take a moment to think about the most basic economics within the question before getting into the details of the answers.*

$$Max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \gamma \ln(h_t + \bar{h})\}$$

$$K_{ct}^{\theta}(A_c L_{ct})^{1-\theta} \geq C_t + I_{Kt}$$

$$K_{t+1} = (1 - \delta_K)K_t + I_{Kt}$$

$$H_{t+1} = (1 - \delta_H)H_t + I_{Ht}$$

$$K_{Ht}^{\theta}(A_H L_{Ht})^{1-\theta} \geq I_{Ht}$$

$$1 \geq L_{ct} + L_{ht}$$

$$K_t \geq K_{Ht} + K_{Ct}$$

$$\bar{h} \geq 0$$

It has been estimated that the price of housing (relative to other consumption goods) has increased by a factor of about 3 since 1970. Some economists have argued that housing prices have increased because of an increase in demand. Some have argued that higher housing prices reflect supply issues. We will use this model to analyze the contributions of these two factors.

The Competitive Equilibrium of the Economy

Each sub-question below is 3 points.

(a) Define a competitive equilibrium. Assume that the numeraire is c.
(b) Derive the first-order necessary conditions.

(c) Show the steady state versions of these conditions.

(d) Provide expressions for the relative price of housing investment, wage, rental rate of capital.

(e) How does the relative price of housing investment potentially changing over time affect the decision to accumulate housing?

(f) Show the demand for housing has an income elasticity greater than 1 when $\bar{h} > 0$. What does $\bar{h} > 0$ imply for household expenditure allocation for a rich versus a poor economy?

Analyzing the Change in the Relative Price

(2) Consider a 1970 steady state in which the relative price of housing is one. Next, consider a 2022 steady state, in which the relative price of housing has increased by a factor of 3.

Each sub-question below is 5 points.

(a) You are free to change any combination of parameter values in the 2022 steady state to see if you can generate this increase in the relative price of housing, e.g., $\gamma_{2022} = 3 * \gamma_{1970}$, $\delta_{2022} = 3 * \delta_{1970}$, etc.

(b) What does this analysis tell you about the importance of supply versus demand factors in understanding why the price of housing rose? In your answer, describe how the parameters you change correspond to "demand" or "supply".

(c) If you find parameter(s) that can generate the relative price increase, what deeper factors might be causing these parameter value changes, and explain why. (For example, a change in preferences, technological innovations, a change in taxes or regulations, etc.)

Part II

10pt Total for Part II

In this problem, we use an overlapping generations model to compare equilibrium outcomes in a monetary economy in which the government prints new money and in which, as with Bitcoin, agents expend real resources to create new money.

The economic environment is as follows:

Time is $t \in \{1, 2, \ldots\}$. The economy is populated by overlapping generations of two-period lived agents. The measure of agents in each generation is equal to one. The utility of the generation born at time $t \geq 1$ is:

$$\log(c_t^y) + \log(c_{t+1}^o),$$

where c_t^y is the consumption when young, and c_{t+1}^o is the consumption when old. The generation born at time t is endowed with y units of perishable consumption goods when young and nothing else. There is also an initial old generation at time $t = 1$, with utility $\log(c_1^o)$. This initial old generation has no endowment of goods but is endowed with $\overline{M}_0$ units of currency.

We first consider equilibrium outcomes in which the government controls the supply of currency and prints new currency. In this case, there is a government that prints currency and distributes it lump-sum to the current old generation. The amount of new currency printed is denoted by $\overline{T}_t$ and is such that money grows at a constant rate. Namely,

$$\overline{M}_t = \overline{M}_{t-1} + \overline{T}_t \text{ and } \overline{T}_t = (\gamma - 1)\overline{M}_{t-1},$$

for some $\gamma \geq 1$. Notice that after receiving the lump sum transfer of currency, the initial old endowment of currency is $\overline{M}_1$. We use a line over the quantities M and T to denote that these are aggregate variables set by government policy.

Markets are competitive and the only asset traded is currency. Denote the real value of one unit of currency (the inverse of the price level) by q_t.

1. (1pt) Describe the utility maximization problem of a young agent of generation t. Include in your description the budget constraints for this agent when young and old. In those budget constraints use the notation M_t (without a line on top) for the quantity of currency that the young agent decides to purchase when young. Also derive the first order conditions of the utility maximization problem for this agent characterizing the

relationship between an optimal choice of consumption when young and old c_t^y, c_{t+1}^o and currency holdings when young M_t to prices q_t and q_{t+1}.

2. (1pt) Define the problem of an old agent in the initial time period $t = 1$. Include in your description the budget constraints for this agent and characterize the optimal choice of consumption for this agent c_1^o as a function of this agent's endowment and the price q_1.

3. (1pt) Define an equilibrium. Here use your solutions to the prior two parts to characterize consumer optimality. Include the feasibility constraint on consumption and the currency market clearing condition.

4. (1pt) Show that there exits a stationary equilibrium in which agents' consumption allocations do not depend on time, that is $c_t^y = c^y$ and $c_t^o = c^o$ for all t, and where the real value of money $q_t\overline{M}_t = q_1\overline{M}_1$ stays constant over time. (In doing so, solve for c^y, c^o, $q_1 M_1$, and the rate of return from holding currency in terms of y and γ.)

5. (1pt) In the stationary equilibrium you characterized in the question above, how does the real value of money depend on γ? Provide economic intuition. That is, explain how the ratio q_{t+1}/q_t in a stationary equilibrium must vary with γ to keep $q_t\overline{M}_t = q_1\overline{M}_1$ constant over time and show then how the ratio q_{t+1}/q_t is related to the real quantity of currency $q_t\overline{M}_t$ in equilibrium.

6. (1pt) When the government prints money, it generates real revenue equal to $q_t\overline{T}_t$ which it rebates to the old when it makes the transfer of currency to them. In the stationary equilibrium you characterized in the question above, how does the real revenue $q_t\overline{T}_t$ depend on γ? Does it increase in γ or decrease in γ or is the relationship between γ and $q_t\overline{T}_t$ non-monotonic? Provide economic intuition.

7. (4 pt total across 3 subparts below) Now change the setting to one in which agents hold crypto currency, and there is no government currency in circulation. Specifically assume that at time t, a young agent can buy new cryptocurrency on the market at price q_t, or produce T_t units of new cryptocurrency to by using $\Gamma(\overline{M}_t, T_t)$ units of her good endowment. In the cost function, $\overline{M}_t$ is the aggregate stock of crypto currency in the economy, including all the new units produced by other young agents, and T_t (without the upper line) is the new currency produced by an individual agent. Note

that this cost of producing new currency depends on the total outstanding stock of currency as is the case with Bitcoin — the problem that one needs to solve to get a new Bitcoin gets more difficult to solve when more Bitcoins have been produced. Assume that the cost function $\Gamma(x, y)$ is strictly increasing in both of its arguments, strictly convex, with $\partial^2\Gamma_{12}/\partial x\partial y > 0$. The new currency produced can be immediately sold on the market at price q_t. Assume in addition that crypto currency depreciates: a fraction δ of the crypto currency purchased in period t is lost in period $t+1$ (e.g. theft by cybercriminals)

(a) (1pt) Define the problem of a young agent of generation t.

(b) (1pt) Derive and interpret the first-order condition with respect to T_t.

(c) (2pt) Suppose that there is an equilibrium in which money grows at the constant rate γ and in which the real value of money $q_t\overline{M}_t$ remains constant over time. Show that, in such an equilibrium, if it exists, it must be the case that $\gamma = 1$. Explain why crypto currency fosters price stability.

Part III

Part 3a

Consider an Armington trade model like the one we studied in class. Consumption in country i is

$$C_i = \left(\sum_{j \in S} q_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution and q_{ji} denotes consumption in country i of the single good produced by country j. Output in country i is given by $A_i L_i$, where A_i is labor productivity and L_i is labor supply in country i. The resource constraint for good i is

$$A_i L_i = \sum_{j \in S} \tau_{ij} q_{ij},$$

where τ_{ij} is the iceberg trade cost to ship a good from i to j. Markets are competitive and trade is balanced.

Equilibrium wages in a trade equilibrium, $\{w_i\}_{i \in S}$, solve

$$w_i L_i = \sum_{j \in S} \lambda_{ij} w_j L_j,$$

where

$$\lambda_{ij} \equiv \frac{p_{ij} q_{ij}}{Y_j} = \left(\frac{p_{ij}}{P_j} \right)^{1-\sigma},$$

where $p_{ij} = \tau_{ij} \frac{w_i}{A_i}$ and $P_j \equiv \left(\sum_i p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. Per-capita consumption in country i is equal to the real wage, w_i/P_i.

Suppose that, starting in equilibrium, productivity in country $i = 1$ rises from $A_i = 1$ to $A_i' = 2$, and all other model parameters remain constant.

1. (2 points) Provide an equation to solve for the change in the real wage in country 1, $(w_1'/P_1')/(w_1/P_1)$ in terms of aggregate productivity in country 1 and the the domestic spending share in country 1 λ_{11} before and after the shock.

2. (2 points) Suppose that country 1 is in autarky ($\tau_{1j} = \tau_{j1} = \infty$) before and after the shock. What is the change in the real wage in country 1?

3. (2 points) Now suppose that country 1 trades with the rest of the world before and after the shock. Suppose also that, in response to the increase in its productivity, unit labor costs fall in country 1 relative to all other countries. That is, w_1/A_1 falls relative to w_j/A_j for $j \neq 1$. Does the domestic spending share in country 1, λ_{11}, rise or fall after the shock? Explain your answer.

4. (2 points) Compare (qualitatively) the change in the real wage in country 1 in response to the increase in its productivity if country 1 is in autarky (as in question 2) versus the one if country 1 trades with the rest of the world (as in question 3). Explain your answer.

Part 3b

Consider a symmetric two country economy with log-linear household flow utility $u_t = \log C_t - L_t$, cash-in-advance (CiA) constraint $P_t C_t = M_t$, and complete international asset markets. Denote foreign country variables with $*$.

1. Show that optimal labor supply together with CiA imply $W_t = M_t$.

2. Argue that with log utility, complete markets imply efficient international risk sharing according to $P_t C_t = \mathcal{E}_t P_t^* C_t^*$, where $\mathcal{E}_t$ is the nominal exchange rate. Show that, together with CiA, this implies $\mathcal{E}_t = M_t / M_t^*$. What is the significance and limitations of this result?

3. Consider firms with desired price $\tilde{P}_t = W_t$ setting prices subject to a Calvo friction with probability of price non-adjustment θ. If $m_t \equiv \log M_t$ follows a random walk, argue that the log-linearized reset price satisfies $\bar{p}_t = m_t$ and price level dynamics satisfies $p_t = \theta p_{t-1} + (1-\theta) m_t$.

4. The log real exchange rate is defined $q_t = e_t + p_t^* - p_t$. What is the meaning of this variable? Show that under the above assumptions, it follows:

$$q_t = \theta q_{t-1} + \theta(e_t - e_{t-1}).$$

   Why θ governs both the persistence and the volatility of the real exchange rate? Does this resolve the PPP puzzle and why?

5. The model above did not make an explicit assumption about producer or local currency pricing (as it implicitly assumed full home bias). Discuss (in words, without algebra) how these two pricing assumptions differ and what are their differential implications for the real exchange rate dynamics? [hint: think about the limiting case without home bias.]