## UCLA

# Department of Economics <br> Ph. D. Preliminary Exam Micro-Economic Theory 

(SPRING 2022)

## Instructions:

- You have 4 hours for the exam
- Answer all the questions. Each section is weighted equally.


## Section A: Price Theory and Market Equilibrium

## 1. Production in Equilibrium

Consider three firms with the following CRS technologies:

- Firm 1 can produce 1 unit of good 2 using 2 units of good 1 ,
- Firm 2 can produce 1 unit of good 3 using 2 units of good 2 ,
- Firm 3 can produce 1 unit of good 3 directly from $d$ units of good 1 .
(a) Suppose that firm 1 and firm 2 produce a positive amount of good 2 and good 3 respectively in equilibrium. What should be the equilibrium prices for good 1,2 , and 3 ? Find the equilibrium prices by normalizing the price of good 1 to 1 .
(b) For which range of $d$ is it possible for firm 1 and firm 2 to produce good 2 and 3 in equilibrium?
(c) Consider a three-good economy with these three firms/technologies and 10 identical consumers, whose preference is given by $u\left(x_{1}, x_{2}, x_{3}\right)=x_{1}^{1 / 3} x_{2}^{1 / 3} x_{3}^{1 / 3}$ and endowment is given by $e=(6,0,0)$. Assume $d=2$ and find a general equilibrium in this economy.


## 2. Revealed Preference

Suppose that the following three price-consumption pairs are observed for some consumer: $\left(\mathbf{p}^{1}, \mathbf{x}^{1}\right)=$ $((2,1),(1,4)),\left(\mathbf{p}^{2}, \mathbf{x}^{2}\right)=((2,3),(2,2)),\left(\mathbf{p}^{3}, \mathbf{x}^{3}\right)=((1,2),(3,1))$. Answer the following questions.
(a) Is there any consumption bundle that is revealed preferred to another consumption bundle among $\left\{\mathrm{x}^{1}, \mathrm{x}^{2}, \mathrm{x}^{3}\right\}$ ? Is there any consumption bundle that is indirectly revealed preferred to another consumption bundle? Find all such pairs of consumption bundles.
(b) Suppose that this consumer's preference is monotone and strictly convex. What can you conclude about this consumer's preference over $\left\{\mathrm{x}^{1}, \mathrm{x}^{2}, \mathrm{x}^{3}\right\}$ ?
(c) One way to evaluate the welfare change from $\mathbf{x}^{2}$ to $\mathbf{x}^{1}$ is to use $e\left(\mathbf{p}^{1}, u\left(\mathbf{x}^{1}\right)\right)-e\left(\mathbf{p}^{1}, u\left(\mathbf{x}^{2}\right)\right)$ (like compensation variation). Another way is to use $e\left(\mathbf{p}^{2}, u\left(\mathbf{x}^{1}\right)\right)-e\left(\mathbf{p}^{2}, u\left(\mathbf{x}^{2}\right)\right)$ (like equivalent variation). Explain why the former is bounded below by $\mathbf{p}^{1} \cdot\left(\mathbf{x}^{1}-\mathbf{x}^{2}\right)=0$ and the latter is bounded above by $\mathbf{p}^{2} \cdot\left(\mathbf{x}^{1}-\mathrm{x}^{2}\right)=4$.

## Section B: Game Theory

## 3. Marriage

Two married players simultaneously choose whether to Cooperate or $D$ efect at times $t=0,1,2, \ldots$. Stage game payoffs are

|  | $C$ | $D$ |
| :--- | :--- | :--- |
| $C$ | $(1,1)$ | $(0,1+g)$ |
| $D$ | $(1+g, 0)$ | $(0,0)$ |

where future payoffs are discounted at rate $\delta$; assume $0<g<\delta<1$.
(a) Show that there is a (subgame perfect) equilibrium where each player always plays $C$ on the equilibrium path, and compute its equilibrium value $v_{C}$.

From now on assume that there is a continuum of players; at any time $t=0,1,2, \ldots$, players are matched in pairs/relationships and at the end of each period either partner in a pair can unilaterally (simultaneously) choose to divorce, ending the relationship. Divorced players get randomly rematched to another player in the next period. ${ }^{1}$ If no partner divorces, the pair continues to the next period. We solve for symmetric anonymous equilibria, where behavior in a given relationship only depends on the history in this relationship.
(b) In the grim-trigger strategy, partners always cooperate, and as soon as one partner defects, both partners divorce. Show that this is not an equilibrium.
(c) In the incubation strategy, in the first period of a relationship both partners defect, and then (irrespective of what happened in the first period) behavior switches to grim-trigger. Show that this behavior constitutes an equilibrium and characterize its equilibrium value $v_{I}$ (at the beginning of the relationship).
(d) In the dating strategy, in the first period partners randomize between defect (prob $q \in(0,1)$ ) and cooperate (prob $1-q$ ). If both players cooperated, behavior switches to grim-trigger; otherwise the relationship ends. Characterize the equilibrium level of $q$ and the resulting equilibrium value $v_{A}$.
(e) Now assume that partners have a common, continuous randomization device: With probabilty $p$ they both play the incubation strategy, and with probability $1-p$ they both play grim-trigger. Characterize the lowest value of $p^{*}$ for which this is an equilibrium, and the corresponding equilibrium value $v_{R}$.
(f) Compare the value from parts (a), (c), (d) and (e); interpret.

[^0]
## Section C: Information Economics

## 4. Efficient Divorce

Two agents are divorcing and must decide how to break up an indivisible asset (e.g. a car). The agents have privately known, IID valuations $\theta_{i} \sim F[\underline{\theta}, \bar{\theta}]$. Agent $i$ 's utility is

$$
u_{i}=\theta_{i} q_{i}-t_{i}
$$

where $q_{i} \in[0,1]$ is the quantity that agent $i$ is awarded (allowing for random allocations), and $t_{i} \in \mathbb{R}$ is the net monetary transfer made by $i$. The default plan is to decide the allocation by flipping a coin, giving each expected utility $\underline{U}\left(\theta_{i}\right)=\theta_{i} / 2$. However, they decide to consult with a mechanism designer to see if they can obtain a more efficient allocation. The designer proposes a symmetric direct revelation mechanism that maps reports of the agents into allocations $\left\langle q_{i}\left(\theta_{1}, \theta_{2}\right), t_{i}\left(\theta_{1}, \theta_{2}\right)\right\rangle$ and is feasible, $\sum q_{i} \leq 1$. Let $Q\left(\theta_{i}\right)=E_{-i}\left[q_{i}\left(\theta_{1}, \theta_{2}\right)\right]$ and $T\left(\theta_{i}\right)=E_{-i}\left[t_{i}\left(\theta_{1}, \theta_{2}\right)\right]$ be the "reduced-form" allocation and transfer, and $U\left(\theta_{i}\right)$ be $i$ 's equilibrium utility. Either agent can opt out of the mechanism and receive their outside option of $\underline{U}\left(\theta_{i}\right)=\theta_{i} / 2$.
(a) Argue that incentive compatibility implies

$$
\begin{array}{rlrl}
U(\theta) & =U\left(\theta^{*}\right)+\int_{\theta^{*}}^{\theta} Q(x) d x & & \text { for } \theta>\theta^{*} \\
& =U\left(\theta^{*}\right)-\int_{\theta}^{\theta^{*}} Q(x) d x & \text { for } \theta<\theta^{*}
\end{array}
$$

for any $\theta^{*} \in[\underline{\theta}, \bar{\theta}]$. What is the corresponding monotonicity condition?
(b) Prove the mechanism is IR iff the type $\theta^{*}=Q^{-1}(1 / 2)$ wishes to participate. ${ }^{2}$
(c) Suppose the designer takes all the money but must ensure the agents get their outside option. Show that when IR binds, its profits are

$$
\Pi=\int_{\theta^{*}}^{\bar{\theta}} Q(\theta) M R(\theta) d F(\theta)+\int_{\underline{\theta}}^{\theta^{*}} Q(\theta) M C(\theta) d F(\theta)-\frac{1}{2} \theta^{*}
$$

where $M R(\theta)=\theta-\frac{1-F(\theta)}{f(\theta)}$ and $M C(\theta)=\theta+\frac{F(\theta)}{f(\theta)}$.
(d) Show that $Q(\theta)=F(\theta)$ in the efficient mechanism. If $\theta \sim U[0,1]$, what is the welfare gain from efficient allocation when compared to the (default) random allocation?
(e) Suppose $\theta \sim U[0,1]$. Assume the designer uses the efficient mechanism. Show that $\Pi>0$. Why does this matter? ${ }^{3}$
(f) Consider implementing the efficient allocation via an all-pay auction. Each agent bids $b_{i}$; the payment goes to the other agent and is paid whether the agent wins or loses. The good goes to the highest bidder. Assume there is a symmetric monotone bidding strategy $b(\theta)$. In a BNE, agent $i$ then gets

$$
u\left(\theta_{i}, \tilde{\theta}_{i}\right)=\theta_{i} \operatorname{Pr}\left(\tilde{\theta}_{i}>\theta_{j}\right)-b\left(\tilde{\theta}_{i}\right)+E\left[b\left(\theta_{j}\right)\right]
$$

when they bid like type $\tilde{\theta}_{i}$. Characterize the equilibrium bidding strategy.

[^1]
[^0]:    ${ }^{1}$ You can assume that new players are added to the game in every period, so that it is possible to rematch divorced players even if nobody else got divorced in the last period.

[^1]:    ${ }^{2}$ Hint: Prove that net utility $U(\theta)-\underline{U}(\theta)$ is minimized at $\theta^{*}$.
    ${ }^{3}$ Hint: Compare with Myerson-Satterthwaite.

