UCLA Department of Economics **Spring 2022**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to answer all questions. All three parts will receive equal weight in your grade independent of the number of questions contained in that part.

You have four hours to complete this exam.

Part I

This part consists of two questions both of which are required.

Question 1

Consider a one-sector growth economy with two types of consumption goods, c_1 (nondurables and services) and c_2 (durable goods). Durable consumption purchases made in period *t*, x_t, contribute to a stock, c_{2t+1}, the services from which provided utility in period *t* + 1. All quantities are expressed in per capita terms and the population, N_t, grows at the rate $\eta - 1$.

A social planner solves the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t N_t [\alpha \log c_{1t} + (1-\alpha)\log c_{2t} + A\log(1-h_t)]$$

subject to

$$c_{1t} + x_t + i_t = \gamma^t k_t^{\theta} h_t^{1-\theta}, \quad \gamma > 1$$

$$N_{t+1} c_{2t+1} = (1-\delta_D) N_t c_{2t} + N_t x_t$$

$$N_{t+1} k_{t+1} = (1-\delta) N_t k_t + N_t i_t$$

$$N_{t+1} = \eta N_t, \quad \eta > 1$$

$$\mathrm{N}_0 = 1,\ \mathrm{c}_{2,0} \text{ and } k_0 \text{ given.}$$

A. Transform this problem into a stationary dynamic program.

B. Fully characterize the steady state growth path for this economy. Be sure that you have sufficient equations to determine the steady state growth path of c_{1t}, c_{2t}, i_t, x_t, k_t, and h_t.

C. Define a *recursive competitive equilibrium* for the stationary version of this economy. Be sure to state the problems solved by households and firms in your decentralized economy.

D. Given initial values for the state variables, are the allocations that would be chosen by the social planner that same as those that satisfy your equilibrium definition? Explain.

Part I Question 2 Each sub-question is worth 5 points. Read all sub-questions first before writing your answers.

Equilibrium with Time Varying Endowments & Heterogeneity

There is a representative type "M" household with one unit of labor input available for use in even periods, and two units of labor input available for use in odd periods. There is a representative type "N" household that has one unit of labor input available in odd periods, and two units of labor input available for use in even periods.

Both households face the following optimization problem:

$$\max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) - \frac{\phi}{2} h_t^2\}$$

subject to the same budget constraint:

$$w_t h_t + r_t k_t + (1-\delta) k_t \geq c_t + k_{t+1}$$

The initial capital stocks are identical in both islands, k_0.

There is an identical representative firm in each economy operating a production technology to produce output:

$$Y_t = A K_t^{\theta} (\gamma^t H_t^{1-\theta}),$$

where $\gamma > 1$.

(i) Assume that type M households populate their own economy (their economy is made up of just them) and type N households populate their own economy (their economy is made up of just them). Define a stationary, competitive equilibrium for these economies, and solve for the stationary first order necessary conditions characterizing the equilibrium. Make sure to include all relevant constraints, including those that may not be binding.

(ii) Does a steady state exist for these two economies? Explain the economic intutition underlying your answer. Briefly describe a numerical algorithm you could use to approximate the equilirium of this economy.

(iii) Suppose we combine the "M" economy and the "N" economy together into a single economy, such that there was always 3 units of labor input available every period. How would your answer to part (ii) change, including potential differences in equilibrium allocations across the "M" and "N" households in even versus odd periods.

(iv) Suppose instead of disliking working, that the household enjoyed leisure time, and that the utility function for each economy was:

$$\max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \phi(T_t - h_t)\},$$

where T_t is the time endowment in period t. Would your answers to part (ii) and part (iii) change? Why or why not? To answer this question you can just modify the first-order conditions and equilibrium definitions you derived earlier as needed.

Part 2

Recall that when we studied heterogeneous firms in Andy's class, we considered two alternative models. In the first model, the Lucas "span of control" model, productivity z was an *attribute of the manager*. In equilibrium, the manager was paid all of the profits earned by the firm. In the second model, with free entry into the creation of firms, z was an *attribute of the firm*. Managers, hired on a competitive labor markets, earned the same compensation at all firms. In this problem, we consider a third model, in which z is an *attribute of the match* between a firm and a manager. To develop this model, consider a version of the Diamond-Mortensen-Pissarides (DMP) model studied in Pierre's class, in which managers must search for firms and bargain over their compensation.

Time is discrete and runs from zero to infinity. The economy is assumed to remain in a steady state, so we suppress reference to the date t on all variables. There is a measure one continuum of infinitely-lived and risk-neutral agents with a common discount factor $\beta \in (0, 1)$. Agents each have one unit of labor per time period and can choose to supply that unit as a production worker in an existing firm, a recruitment worker spending time searching for a manager on behalf of a new firm, or searching to become a manager. The labor market for production and recruitment workers is perfectly competitive, with a wage W. The labor market for managers is a DMP search-and-matching market. Agents who are searching to become a manager in a new firm receive an unemployment benefit bW, where $b \in (0, 1)$. Note that this benefit is directly proportional to the equilibrium wage for production and recruitment workers.

There is a large number of potential firms who can create vacancies for a new managers instantly. To maintain a vacancy, a firm must hire c units of recruitment worker labor time at wage W. Hence the vacancy cost for a new firm searching for a manager is cW per period. As in Pierre's notes we let θ denote the vacancy to unemployment ratio. With this notation, the vacancy-filling probability of a firm is $q(\theta)$, for some decreasing function q. Correspondingly, the job-finding probability of a manager is $\theta q(\theta)$.

When an unemployed manager and a vacant firm meet, they draw an idiosyncratic match productivity z which is independently and identically distributed according to some CDF $F(z)$. If the firm hires the manager, it can produce output by hiring, in addition, l production workers at wage W. The production technology is

$$Y = z^{1-\nu} l^{\nu},$$

for some $\nu \in (0, 1)$. If there are gains from trade between the new firm and the manager, the firm hires the manager at a per-period wage $\Omega(z)$, determined so that the manager receives a fraction $\phi \in (0, 1)$ of the surplus. After a match is formed, it may be exogenously destroyed in subsequent periods. Every period, the probability of match destruction is $\delta \in (0, 1)$.

Before diving into the questions below, notice that the model is very close to the models of heterogeneous firms that we studied in Andy's class except that productivity z is not solely an attribute of the manager or the firm, but instead is determined by the quality of the match between the manager and the firm. The market for manager is very close to the DMP model in Pierre's class and in PS02, but with three key differences. First, the measure of agents on the DMP market for manager is endogenous. Second the recruiting costs and unemployment benefits are proportional to W. Third, the production function has two inputs, managers and production workers.

1. (1pt) Consider a firm who has hired a manager with idiosyncratic match quality z. This firm hires production labor on a competitive market for this labor to maximize profits $z^{1-\nu}l^{\nu} - Wl$. Calculate the optimal labor demand $l(z)$ of the firm and show that firm production employment, output, and profits are given by:

$$l(z) = z \left(\frac{\nu}{W}\right)^{1/(1-\nu)} \tag{1}$$

$$Y(z) = z \left(\frac{\nu}{W}\right)^{\nu/(1-\nu)} \tag{2}$$

$$Y(z) - Wl(z) = (1-\nu)Y(z). \tag{3}$$

2. (1pt) Write the Bellman equation for the value of an unemployed manager, V_U, for an employed manager if the idiosyncratic match quality is z, $V_E(z)$, for a new firm posting a vacancy, Π_V, and a for a filled firm with productivity z, $\Pi_F(z)$. Remember that, when a firm and a manager meet, they draw a random z and choose whether to form a match.

3. (1pt) Let the surplus in a filled match between a firm and a manager be $\Sigma(z) \equiv \Pi_F(z) + V_E(z) - \Pi_V - V_U$. Write the two surplus sharing equations that link the net utility of the manager and of the firm, $V_E(z) - V_U$ and $\Pi_F(z) - \Pi_V$, to the surplus $\Sigma(z)$.

4. (1pt) Show that the Bellman equation for the surplus can be written as:

$$\begin{aligned}(1-\beta)\Sigma(z) =& (1-\nu)Y(z) - bW + cW \\ & - \beta q(\theta)(1-\phi)\int \max\{\Sigma(z),0\}\,dF(z) - \beta\theta q(\theta)\phi\int \max\{\Sigma(z),0\}\,dF(z).\end{aligned}$$

5. (1pt) Argue that since firms can freely enter and create vacancies, it follows that $\Pi_V = 0$. Argue that since agents can choose to be workers or managers, $(1-\beta)V_U = W$. Using these two conditions, together with the surplus sharing equations, show that

$$\theta = \frac{1-b}{c}\frac{1-\phi}{\phi}.$$

Discuss the comparative statics of θ with respect to b, c and ϕ.

6. (1pt) After imposing the free entry condition $\Pi_V = 0$, show that:

$$(1-\beta(1-\delta))\Sigma(z) = (1-\nu)Y(z) - bW - \beta\theta q(\theta)\phi\int \max\{\Sigma(z),0\}\,dF(z). \qquad (4)$$

Argue that there is a reservation productivity $z^\star$ such that a firm and a manager form a match when if $z \geq z^\star$. Argue that the smallest firm that one would see in the model (in terms of output and employment) is a firm with productivity index $z^\star$.

7. (1pt) Argue that, at $z^\star$, $(1-\nu)Y(z^\star) = \Omega(z^\star) = W$. Explain this result. Does the owner of a firm with productivity index z^* earn any money from operating the firm? Does the manager of this firm earn any more than he or she could as a worker?

8. (1pt) Describe the implications of this model for the relationship between firm size and the compensation of the firm's manager. Specifically, consider two operating firms with productivity indices z and $z^\star$, with $z > z^\star$. What is the relationship between the difference in firm sizes measured in terms of output, $Y(z) - Y(z^\star)$, and managerial compensation for these firms relative to that of workers, $\Omega(z) - \Omega(z^\star) = \Omega(z) - W$. Would managers of very large firms be paid a lot of money in this model?

Would you say that their compensation was determined by skill or luck? (This last question is an open-ended question for you to discuss if you have time)

Part 3

Answer both questions to the extent possible; split your hour equally between the two. Provide an intuitive explanation if you cannot derive the result formally.

1 Armington model with endogenous labor supply

Consider a multi-country Armington trade model like the one we studied in class but in which labor supply in each country is endogenous.

For given labor supply L_i in each country, the model is exactly like the Armington model we studied in class. In that model, consumption in country i is

$$C_i = \left(\sum_{j \in S} q_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution and q_{ji} denotes consumption in country i of the single good produced by country j. Output in country i is given by $A_i L_i$, where A_i is labor productivity. The resource constraint for good i is

$$A_i L_i = \sum_{j \in S} \tau_{ij} q_{ij},$$

where τ_{ij} is the iceberg trade cost to ship a good from i to j. Markets are competitive and trade is balanced.

Given labor supplies by country $\{L_i\}_{i \in S}$, equilibrium wages in a trade equilibrium, $\{w_i\}_{i \in S}$, solve

$$w_i L_i = \sum_{j \in S} \lambda_{ij} w_j L_j,$$

where

$$\lambda_{ij} \equiv \frac{X_{ij}}{Y_j} = \left(\frac{p_{ij}}{P_j} \right)^{1-\sigma},$$

where $p_{ij} = \tau_{ij} \frac{w_i}{A_i}$ and $P_j \equiv \left(\sum_i p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. Aggregate consumption in country i is given by $C_i = \frac{w_i L_i}{P_i}$.

To model the endogenous determination of labor supply, we assume that the representative household in country $i \in S$ maximizes utility

$$U_i(C_i, L_i) = \log\left(C_i - \frac{\phi_0}{1+\phi} L_i^{1+\phi}\right)$$

subject to the budget constraint $C_i P_i = w_i L_i$.

1. Provide an equation to solve for labor supply L_i in country i as a function of the real wage w_i/P_i. Show that when $\phi \to \infty$, labor supply is inelastic with respect to the real wage w_i/P_i.

   Consider an initial trade equilibrium in which country i's domestic spending share is $\lambda_{ii} < 1$. Suppose that country i moves to autarky, in which bilateral trade costs are $\tau_{ij} = \tau_{ji} = \infty$ for $i \neq j$ and the domestic spending share is $\lambda_{ii} = 1$. Productivities and other parameters remain unchanged.

2. Write a system of equations to solve for the change in the real wage, labor supply, and consumption in country i, given λ_{ii} in the initial trade equilibrium and other model parameters.

3. How do your answers in 2. compare to the case of inelastic labor supply ($\phi \to \infty$)?

2 New-Keynesian Model

Consider an infinite-horizon economy with households maximizing:

$$\max_{\{C_t, L_t, \{B_{t+1}\}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log C_t - L_t]$$

subject to

$$P_t C_t + B_{t+1} \leq R_t B_t + W_t L_t + \Pi_t,$$

The government controls aggregate demand by supplying money for nominal transactions, $P_t C_t = M_t$. Firms operate a linear production function, $Y_t = A_t L_t$, and set prices subject to a Calvo frictions with a probability of price adjustment $1 - \theta$.

1. Prove that equilibrium wages satisfy $W_t = M_t$.

2. Assuming that firms face constant-elasticity demand curves, $C_{it} = (P_{it}/P_t)^{-\eta} C_t$, set up the firm's price setting problem and prove that the optimal reset price is given by:

$$\bar{P}_t = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\theta)^j P_{t+j}^{\eta-1} MC_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\theta)^j P_{t+j}^{\eta-1}},$$

   where marginal cost $MC_t = W_t / A_t$. Explain this result.

3. Prove that the log-linearized version of the optimal reset price is given by:

$$\bar{p}_t = (1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t mc_{t+j},$$

   and argue why the dynamics of the price level satisfies:

$$p_t = \theta p_{t-1} + (1 - \theta) \bar{p}_t.$$

4. Derive the Phillips curve for inflation $\pi_t = \Delta p_t$:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda (mc_t - p_t), \qquad \lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}.$$

   Explain the significance of this equilibrium condition.

5. If m_t follows a random walk and there are no productivity shocks ($a_t = 0$), prove that:

$$p_t = \theta p_{t-1} + (1 - \theta) m_t$$

   and π_t follows an AR(1) with persistence θ and iid innovation $(1 - \theta)\Delta m_t$. Why does θ increase the persistence, but reduce the volatility of the inflation process? Derive the impulse response of aggregate output y_t to a monetary shock m_t.