UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2021)

Instructions:

- You have **6** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

1. Optimal Consumption

Consider a consumer whose preference on three goods can be represented by the following utility function.

$$u(\mathbf{x}) = \sqrt{x_1 x_2} + x_3$$

Let $p_\ell \geq 0$ be the price of good $\ell = 1, 2, 3$. Normalize the price of good 3 to 1 in the following questions.

(a) Suppose that this consumer can spend $600 on either good 1 or 2 (but not on good 3). Derive her optimal consumption of good 1 and good 2 as a function of (p_1, p_2).

(b) Write down the utility maximization problem for this consumer with wealth $w > 0$. Characterize the range of (p_1, p_2, w) where she consumes a positive amount of good 3 as a part of her optimal consumption plan.

(c) Provide a complete description of this consumer's Marshallian demand functions $x_\ell((p_1, p_2), w), \ell = 1, 2, 3$.

(d) Derive this consumer's Hicksian demand functions $h_\ell((p_1, p_2), \underline{u}), \ell = 1, 2, 3$.

2. General Equilibrium

We consider an economy with two factors: labor and land. There are N consumers in this economy. Each consumer is initially endowed with d unit of lands and 24 hr of time which he can use either for leisure or for working. There is only one consumption good that can be produced using labor and land. If consumer i consumes x_i amount of good and spends $24 - \ell_i$ hours as leisure time (which means that he works for ℓ_i hours), then his utility is given by $\ln x_i + \ln(24 - \ell_i)$. When working for ℓ_i hours, his income is $w\ell_i + rd$, where w is the hourly wage and r is the rental price of land respectively. There is a producer who can produce $D^{0.5}L^{0.5}$ of the consumption good using D unit of lands and L hours of labor. Normalize the price of consumption good to 1. Answer the following questions.

(a) Is it possible for consumers to spend 24 hours for pure leisure (not working at all) in equilibrium for some parameter value of $d > 0$? Discuss. Maintain the assumption that every consumer owns exactly the same amount of land.

(b) Explain why the producer's profit must be 0 in equilibrium. Derive the condition on the wage w and rental price r that is implied by the 0 profit condition.

(c) Find the general equilibrium price (w^*, r^*) and the optimal consumption of good and leisure in equilibrium.

3. A Game of Tag

When not on a case, Holmes and Watson amuse themselves by playing tag in the London Tube (subway). Watson gets on the first train at the Baker Street station, Holmes gets on the next train.

Watson can get off at the Liverpool station (which comes first) or at Bow Church (which comes last). Holmes does not see the choice Watson makes. If Watson and Holmes get off at different stations, then Holmes will never tag Watson. If they both get off at the same station, and the station is not crowded, then Holmes will tag Watson all the time, but if the station is crowded, Holmes will only tag Watson one fourth of the time. Liverpool will be crowded half the time; Bow Church is never crowded. Holmes has inside information so he knows whether Liverpool is crowded, but Watson does not.

If Holmes wins (tags Watson) then Watson pays Holmes 4 pounds; otherwise Holmes pays Watson 8 pounds. Holmes and Watson can ride the Tube for free (they have special passes) but it costs each of them 2 pounds to go to Bow Church. [Baker Street and Liverpool are in Zone 1; Bow Church is in Zone 2, which is an extra charge.]

(a) Draw the extensive form of the game.
[Model Watson and Holmes as each having two actions: get off at Liverpool or get off at Bow Church.]

(b) Find all the Perfect Bayesian Equilibria of this game (in mixed behavioral strategies).

[Holmes and Watson are risk neutral and so maximize expected monetary payoff.]

4. Repeated Game

Consider the game G below:

	L	M	R
T	6,1	0,0	0,0
M	0,0	5,5	0,12
B	0,0	12,0	1,6

Now consider the infinitely repeated version of this game $G^{\infty}(\delta)$, where players discount future payoffs at the constant rate $\delta \in (0, 1)$ per period. [Assume both players use pure strategies.]

Before play starts, ROW and COL are tempted to agree to play (M, M) forever, and punish deviations as necessary. But then ROW suggests that, instead, they should alternate between (B, M) and (M, R), and punish deviations as necessary.

(a) For which values of δ is there a SGPE of $G^{\infty}(\delta)$ in which play along the equilibrium path is (M, M) forever? [You must specify the SGPE strategies.]

(b) For which values of δ is there a SGPE of $G^{\infty}(\delta)$ in which play along the equilibrium path alternates between (B, M) and (M, R)? [You must specify the SGPE strategies.]

(c) For which values of δ do both players prefer to play (M, M) forever?

5. Limited Liability

A principal employs an agent to undertake a project that either succeeds and produces output $y = V$, or fails and produces output $y = 0$. If the agent exerts effort $e \in [0, 1]$ at cost $C(e) = e^2/2$, the project succeeds with probability $Pr(y = V) = e$. Assume $V \in (0, 1)$.

(a) Suppose a social planner can choose effort to maximize expected welfare, $E[y] - C(e)$. What is their choice of effort?

Suppose the principal must incentivize the agent to choose effort. She pays the agent W_1 if the project succeeds and W_0 if it fails. The principal's profit equals the output minus the wage, $\Pi = y - W$. The agent is risk-neutral and has utility $u = W - C(e)$. Assume the agent is protected by limited liability, so $W_1, W_0 \geq 0$.[1]

(b) Given contract (W_1, W_0), what effort will the agent choose?

The principal seeks to choose wages and an effort recommendation, (W_1, W_0, e) to maximize her profits subject to the agent's first order condition (ICFOC) and $W_1, W_0 \geq 0$ (LL).

(c) Show that $W_0 = 0$.

(d) What is the optimal W_1?

(e) How does the optimal effort compare with the social planner's effort in (a)? What is the reason for this difference?

[1] In this problem we ignore the agent's outside option. The limited liability means that their utility is always non-negative.

6. Auctions with Heterogeneous Quality

A seller has two goods to sell with qualities q^H and q^L, where $q^H > q^L$. There are $N \geq 3$ bidders with IID types $\theta_i \sim F[0,1]$ who have unit demand.[2] Suppose agent i reports $\tilde{\theta}_i$ and let $\tilde{\theta}$ be the vector of reports. A direct mechanism describes the probability that an agent is awarded the high-quality good $p_i^H(\tilde{\theta}) \in [0,1]$, the probability he is awarded the low-quality good $p_i^L(\tilde{\theta}) \in [0,1]$, and the transfer to the seller $t_i(\tilde{\theta}) \in \mathbb{R}$.

Payoffs are as follows. An agent with type θ_i obtains utility,

$$u_i = \left(p_i^H q^H + p_i^L q^L\right) \theta_i - t_i$$

where $p_i^H + p_i^L \leq 1$, since the agent only wants one unit. The seller gets all the payments and has reserve value of zero. Thus her profit is

$$\Pi = \sum_i t_i$$

(a) What is the welfare-maximizing allocation?

(b) Suppose we wish to implement the welfare-maximizing allocation in dominant strategies. What are the payments under the pivot mechanism?[3]

(c) Assume $MR(\theta_i) = \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}$ is increasing in θ_i. What is the profit-maximizing allocation?

[2] As an application, suppose the New York Times is selling display advertising. A single page features a big ad and a small one.

[3] Recall, in an auction, the pivot mechanism is a special case of the VCG mechanism where the lowest type makes no payment.