UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2020)

1. Choice by Satisficing: Let $X = \{x_1, x_2, ..., x_N\}$ be a set of N goods and $\mathcal{B}$ be the collection of all non-empty subsets of X. There is a decision maker (DM) whose utility function is given by $u : X \to \Re$. Assume that this DM has a strict preference on X, so u assigns a different number to different goods.

DM is not a utility maximizer. DM has a reference utility level u^*. Given a problem $B = \left\{x_{i(1)}, ..., x_{i(|B|)}\right\}$, where $i(k)$ is the kth smallest index within B (for example $x_{i(1)} = x_2$ for $B = \{x_2, x_3, x_6\}$), DM follows the following decision procedure to make a choice.

- Step 1: If $u(x_{i(1)}) \geq u^*$, then DM chooses $x_{i(1)}$. If $u(x_{i(1)}) < u^*$, then go to Step 2.
- Step k: If $u(x_{i(k)}) \geq u^*$, then DM chooses $x_{i(k)}$. If $u(x_{i(k)}) < u^*$, then go to Step $k+1$.
- Step $|B|$: DM chooses $x_{i(|B|)}$ (the good with the largest index in B).

Given $u : X \to \Re$ and u^*, the above procedure generates a choice function $\tilde{C}$ on $\mathcal{B}$ given by $\tilde{C}(B) = x_j$, where $j = \arg\min \{i : x_i \in B,\ u(x_i) \geq u^*\}$ or, if this set is empty, $j = \arg\max \{i : x_i \in B\}$.

Answer the following questions.

(a) Suppose that $B = \{x_2, x_3, x_5\}$ and $u(x_3) > u(x_5) > u^* > u(x_2)$. Which good would this DM choose from B?

(b) Prove that $\tilde{C}$ satisfies WARP (Weak Axiom of Revealed Preference).

(c) Is there any complete and transitive preference $\succeq$ on X such that $\tilde{C} = C_{\succeq}$ on $\mathcal{B}$? If so, find such a preference. If not, prove that there is no such preference.

For the next question, consider a DM with the same utility function u and a decreasing sequence of reference levels $u_1^* > u_2^* > ... > u_N^*$, who follows the following decision procedure:

- Step 1: If $u(x_{i(1)}) \geq u_1^*$, then DM chooses $x_{i(1)}$. If $u(x_{i(1)}) < u_1^*$, then go to Step 2.
- Step 2: If $\max\left\{u(x_{i(1)}), u(x_{i(2)})\right\} \geq u_2^*$, then DM chooses the better one from $x_{i(1)}$ and $x_{i(2)}$ (DM gets impatient and lowers the standard). Otherwise, go to Step 3.
- Step k: If $\max\left\{u\left(x_{i(1)}\right), \cdots, u\left(x_{i(k)}\right)\right\} \geq u_k^*$, then DM chooses $x_j \in B$, where $u(x_j) = \max\left\{u\left(x_{i(1)}\right), \cdots, u\left(x_{i(k)}\right)\right\}$. Otherwise, go to Step $k+1$.
- Step $|B|$: DM chooses x_j where $x_j = \arg\max_{x_i \in B} u(x_i)$.

(d) The above procedure defines a different choice function $\widehat{C}$ on $\mathcal{B}$. Show by an example that $\widehat{C}$ does not satisfy WARP.

2. Common Value Auction: Consider a common value first price auction for a single object, with two bidders. The signals space is $S = [0,1] \times [0,1]$; signals are uniformly distributed on S. If Nature draws the signal (s_1, s_2) then Bidder 1 learns s_1 and Bidder 2 learns s_2 and the true common value of the object is $s_1 + s_2$.

You are Bidder 1.

(a) If you receive the signal s_1, show that your expected value *conditional on having the higher signal* is $(3/2)s_1$.

(b) Suppose Bidder 2 follows the strategy of bidding half of his expected value *conditional on having the higher signal*; i.e. he follows the strategy $B_2^*(s_2) = (3/4)s_2$. What is your best response B_1^* to B_2^*?

(c) Is the pair (B_1^*, B_2^*) a (Bayesian) Nash Equilibrium?

(d) Find a *symmetric* Bayesian Nash Equilibium; i.e. a strategy/function B such that (B, B) is a (Bayesian) Nash Equilibrium.

3. Assembling Venture Capital: An entrepreneur has a project that costs I and yields return $I + A$, but she has no money. The required investment $I \sim F[0, \bar{I}]$ is a random variable and initially unknown to both parties. An investor has money x that it can invest in the project.

The game is as follows. First, the entrepreneur offers return r to the investor. Second, the investor chooses whether to accept the contract ($y = 1$) or reject the contract ($y = 0$). If it accepts, it invests x into the project.[1] If it rejects, it gets $x(1+\theta)$, where $\theta > 0$ measures the opportunity cost of investment. Third, the required investment I is publicly realized.

Payoffs are as follows. If the agent invests and $x \geq I$, the project succeeds and the investor gets $(1 + r)x$. If $x < I$, the investor gets its money back, x. In expectation, if the investor accepts the contract, it gets profit

$$\Pi(x) = (1 + r)xF(x) + x(1 - F(x))$$

The entrepreneur gets

$$U(x) = F(x)(A - rx)$$

Since x is fixed, she thus wishes to minimize the cost of attracting the investment.

(a) The entrepreneur chooses the return r to maximize her utility subject to the investor accepting the contract (i.e. the entrepreneur has the bargaining power). What is the optimal return r? Given the optimal r, what is the entrepreneur's utility?

Now, suppose that there are two investors with funds x_1 and x_2, where $x_1 > x_2$. The game is as follows. First, the entrepreneur offers return r_i to investor $i \in \{1, 2\}$. Second, the investors *simultaneously* choose whether to accept the contract ($y_i = 1$) or reject the contract ($y_i = 0$). If i accepts, it invests x_i into the project. Third, the required investment I is publicly realized. If the money raised exceeds the required investment, $x_1y_1 + x_2y_2 \geq I$, the project succeeds and an investor gets $(1 + r_i)x_i$. Otherwise, an investor gets its money back, x_i. In expectation, if investor i accepts the contract, it gets profit

$$\Pi_i(x_i) = (1 + r_i)x_iF(x_i + x_jy_j) + x_i(1 - F(x_i + x_jy_j))$$

for $j \neq i$. As above, the opportunity cost of investing is $(1 + \theta)x_i$. Meanwhile, the entrepreneur then wishes to minimize the cost of attracting the investment.

(b) The entrepreneur chooses the return r_i for $i \in \{1, 2\}$. Suppose we want $(y_1, y_2) = (1, 1)$ to be a Nash Equilibrium of the investment game. What are the optimal interest rates (r_1, r_2)? Comparing this with part (a) how does this addition of the second agent affect the return?

(c) The entrepreneur chooses a common return r. Suppose we want $(y_1, y_2) = (1, 1)$ to be the *unique* Nash Equilibrium of the investment game. What is the optimal r?

(d) The entrepreneur chooses the return r_i for $i \in \{1, 2\}$. Suppose we want $(y_1, y_2) = (1, 1)$ to be the *unique* Nash Equilibrium of the investment game. Suppose that $1/F(x)$ is convex.[2] What are the optimal returns, (r_1, r_2)? Which investor gets the better return?

[1] Throughout, we assume it's optimal for the investor(s) to invest all of their money. This is optimal if returns A are high, or wealth x is small.

[2] Useful fact: If $\phi(x)$ is convex then $[\phi(x_1)x_1 - \phi(x_2)x_2] \leq \phi(x_1 + x_2)\,[x_1 - x_2]$ for $x_1 > x_2$.