UCLA Department of Economics **Spring 2021**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to answer all questions. All three parts will receive equal weight in your grade independent of the number of questions contained in that part.

You have six hours to complete this exam. Please keep your camera on the whole time and let the proctor know when you need to leave the room indicating how long you will be gone.

The exam is open book, so you can consult your notes and other materials.

Please send Chiara your completed exams via email and she will anonymize them for the faculty grading the exam. Please do **not** include your names on the text of your submissions. Either typed **or** handwritten and scanned answers are okay as long as they are legible.

Part I

This part consists of two questions.

Question 1

Consider the following social planning problem:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - h_{1t} - h_{2t}) \qquad \text{where } 0 < \beta < 1$$

subject to

$$c_t = F^1(k_{1t}, h_{1t})$$

$$k_{t+1} = F^2(k_{2t}, h_{2t})$$

$$k_t = k_{1t} + k_{2t}$$

$$k_0 \text{ given.}$$

Here, the function U, F^1 and F^2 have all the usual properties. Assume that capital can be freely allocated across the two sectors.

a. Write this maximization problem as a dynamic program.

b. Define a *recursive competitive equilibrium* for this economy. Be sure to state the problems solved by households and firms in your definition and define any additional variables you introduce.

c. Provide a set of equations that characterizes the steady state of this economy.

Question 2

There are two types of people in the economy, types 1 and 2, and the number of each type is normalized to 1. Each person has one unit of time that they can use to work in the market or as leisure. A type "i" person has k_i units of capital that does not depreciate nor can it be accumulated. Total capital, k,is given by $k = k_1 + k_2$

Type 1's utility function is given by:

$$\max \sum_{t=0}^{\infty} \beta^t (c_{1t} - \phi_1 \frac{h_{1t}^2}{2})$$

Type 2's maximization problem is given by:

$$\max \sum_{t=0}^{\infty} \beta^t (\frac{(c_{2t} - \bar{c})^2}{2} - \phi_2 h_{2t})$$

There is an aggregate production technology, in which output is produced using:

$$\begin{aligned} y_t &= A h_{1t}^{\theta_1} h_{2t}^{\theta_2} k^{(1-\theta_1-\theta_2)}, \\ y_t &= c_{1t} + c_{2t} \end{aligned}$$

A.Define a competitive equilibrium

B. Solve for the equations characterizing the equilibrium prices and quantities, and solve for those prices and quantities if closed form solutions exist. Describe the economic intuition behind these first order conditions and the solutions (if solutions for any of the endogenous variables exist).

C. Suppose that the production technology changed to the following

$$A h_{1t}^{\theta_1} h_{2t}^{\theta_2} (B k^{(1-\theta_1-\theta_2)}), B > 0$$

where B is a capital-specific technology term. Explain how this term affects the rental prices of the two types of labor, the rental price of capital, and the market price of capital.

D. How would a social planner, who values the utility of each type the same, implement the competitive equilibrium allocations? (You can solve the social planner problem if you like, though it is not required.)

Part 2 *This part consists of one question.*

(10 points)

In this problem, we examine how idiosyncratic risk alters entrepreneurs' decisions to start a firm and we explore the general equilibrium implications of this interaction between risk and business formation.

We consider a Lucas span-of-control model. There is a measure one of agents, each endowed with one unit of labor. Each agent also has expected productivity as a manager of $x \in [1, \infty)$, where x is idiosyncratic to each person and is drawn independently across agents from a distribution with cumulative distribution function $G(x)$ and strictly positive density $g(x)$. Each agent, knowing its own x, chooses whether to become a manager or a worker with knowledge of their own x. After agents choose to manage a firm, they learn their true productivity

$$z = x \times y$$

where y is a random variable with <u>mean one</u>. The draw of y is independent of x and across agents. The decision to become a manager is thus risky because agents do not know their true productivity as a manager when they make their decision to manage a firm: they know their agent-specific x but do not know their agent-specific y.

A firm with a manager with productivity z that hires ℓ units of labor produces output

$$z^{1-\nu} \ell^{\nu} \tag{1}$$

for some fixed value of $\nu \in (0, 1)$.

We consider equilibrium in this model in two stages: a first stage in which people choose whether to be a manager or a worker and then a second stage in which managers hire workers and produce output. We guess and later verify that, in equilibrium there is some cutoff $x^\star$ such that agents choose to be workers if $x < x^\star$, and choose to be managers if $x \geq x^\star$.

Part A: labor market clearing in the second stage. In the second stage, managers with productivity z take the wage rate W as given and choose labor ℓ to maximize profits

$$\pi(z, W) = \max_{\ell} z^{1-\nu} \ell^{\nu} - W \ell \tag{2}$$

A.1. (1pt) Show that the labor demand of a firm with productivity z is:

$$\ell(z, W) = z \left(\frac{W}{\nu} \right)^{-\frac{1}{1-\nu}}$$

A.2. (1pt) Keeping in mind that $z = x \times y$ and that y has a mean of 1, show that the aggregate labor demand is:

$$L^d(x^\star, W) = \int_{x^\star}^{\infty} x \left(\frac{W}{\nu}\right)^{-\frac{1}{1-\nu}} g(x)\, dx.$$

A.3. (1pt) Argue that the aggregate labor supply is $L^s(x^\star, W) = G(x^\star)$.

A.4. (1pt) Argue that the market clearing condition for labor,

$$L^d(x^\star, W) = L^s(x^\star, W),$$

defines a decreasing relationship between the wage rate, W, and the participation cutoff, $x^\star$. What is the economic logic explaining that the relationship is decreasing? Draw that relationship (the "labor market clearing curve") in a diagram with wage on the x-axis and participation cutoff on the y-axis.

Part B: participation in the first stage. Suppose agents have a concave utility over their consumption. The consumption of a worker is equal to the wage, W, while the consumption of a manager is equal to its profit, $\pi(z, W)$.

B.1. (1pt) Show that the profit of a manager with productivity z is:

$$\pi(z, W) = (1-\nu)\, z \left(\frac{W}{\nu}\right)^{-\frac{\nu}{1-\nu}}.$$

B.2. (1pt) Recall that $z = x \times y$ and consider the expected utility of a manager, before y realizes:

$$\mathbb{E}\left[U\left(\pi(x\, y, W)\right)\right],$$

where the expectation is taken with respect to the distribution of y. Show that the expected utility of a manager decreases with W and decreases when the distribution of y becomes riskier, in the sense of second-order stochastic dominance.

B.2. (1pt) Argue that, in an equilibrium, the manager at the cutoff $x^\star$ must be indifferent between becoming a worker and a manager:

$$\mathbb{E}\left[U\left(\pi(x^\star\, y, W)\right)\right] = U(W). \tag{3}$$

B.3 (1pt) Argue that (3) defines an upward slopping relationship between W and $x^\star$. What is the economic logic explaining that the relationship is increasing? Draw that relationship (the "optimal participation curve") in a diagram with wage on the x-axis and participation cutoff on the y-axis.

B.4 (1pt) Show that the optimal participation curve shifts up in the diagram when the distribution of y becomes riskier, in the sense of second-order stochastic dominance.

Part C: Equilibrium participation and idiosyncratic risk.

C.1. (1pt) Draw the labor market participation curve and the optimal participation curve in the same diagram. What happens to the wage rate and the cutoff participation when the distribution of y becomes riskier? Explain the economic logic behind this result.

C.2 (1pt) Suppose that managers can insure against all idiosyncratic risk y. Argue that in this case manager consumption must be equal to their expected profits. Argue that the equilibrium participation in this case is the same as the one in an economy in which y is deterministic and equal to one.

Part 3

This part consists of two questions.

Question 1: Try to answer all questions below, but a complete solution is not expected. Formal proofs are only needed when requested explicitly, however, intuitive explanations can substitute for formal proofs.

Consider an infinite-horizon economy with households maximizing:

$$\max_{\{C_t, L_t, \{B_{t+1}\}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log C_t - L_t]$$

subject to

$$P_t C_t + \sum_{s_{t+1}|s^t} Q_{t+1} B_{t+1} \leq B_t + W_t L_t + \Pi_t,$$

where $\{B_{t+1}\}$ is a full set of Arrow securities. The government controls aggregate demand by supplying money for nominal transactions, $P_t C_t = M_t$. Firms operate a linear production function, $Y_t = A_t L_t$, and set prices subject to a Calvo frictions with a probability of price adjustment $1 - \theta$.

1. Prove that equilibrium wages satisfy $W_t = M_t$. Explain why in a symmetric open economy the nominal exchange rate satisfied $\mathcal{E}_t = M_t / M_t^*$.

2. Assuming that firms face constant-elasticity demand curves, $C_{it} = (P_{it}/P_t)^{-\varrho} C_t$, set up the firm's price setting problem and prove that the optimal reset price is given by:

$$\bar{P}_t = \frac{\varrho}{\varrho - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{C_{i,t+j}}{M_{t+j}} MC_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{C_{i,t+j}}{M_{t+j}}},$$

   where marginal cost $MC_t = W_t / A_t$. Explain this result.

3. Explain why the log-linearized version of the optimal reset price is given by:

$$\bar{p}_t = (1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t mc_{t+j},$$

   and why dynamics of the price level satisfy:

$$p_t = \theta p_{t-1} + (1 - \theta) \bar{p}_t.$$

4. Derive the Phillips curve for inflation $\pi_t = \Delta p_t$:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda (mc_t - p_t), \qquad \lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}.$$

   Explain the significance of this equilibrium condition.

5. If m_t follows a random walk and there are no productivity shocks ($a_t = 0$), prove that:

$$p_t = \theta p_{t-1} + (1 - \theta) m_t$$

   and π_t follows an AR(1) with persistence θ and iid innovation $(1 - \theta)\Delta m_t$.

6. Explain why in an open economy version, the real exchange rate q_t follows an AR(1) with persistence θ and iid innovation $\theta \Delta e_t = \theta(\Delta m_t - \Delta m_t^*)$.

Consider a closed economy model with homogeneous monopolistically competitive firms and endogenous entry. There is a mass of agents, $L > 0$, each of which supplies a unit of labor inelastically. The representative household has CES preferences over differentiated varieties indexed by ω given by

$$U = \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $q(\omega)$ is the quantity consumed of variety ω and Ω is the set of varieties with positive consumption. Consumpion (or utility) per capita is $u \equiv \frac{U}{L}$.

In order to enter and be able to produce a new variety ω, a firm must hire f workers. After the firm has payed the fixed cost, its productivity is A, i.e. it must employ $\frac{1}{A}$ units of labor per unit of output. A is equal across firms.

The labor resource constraint is

$$L = M\left(\frac{q}{A} + f\right)$$

where M is the mass of entering firms, and where I have used the result that, in equilibrium, $q = q(\omega)$ across all varieties that are produced.

We assume that firms are monopolistically competitive and charge a price which is a fixed markup over marginal cost

$$p(\omega) = \frac{\sigma}{\sigma - 1}\frac{1}{A}$$

where we have normalized the wage to 1. Profits of each firm are given by

$$\pi(\omega) \equiv \left(p(\omega) - \frac{1}{A}\right) q(\omega) - f$$

The free entry condition implies that

$$\pi(\omega) = 0$$

1. Show that the equilibrium mass of entering firms is

   $$M = \frac{L}{\sigma f}$$

   and output per variety is

   $$q(\omega) = q = Af(\sigma - 1)$$

2. Suppose that productivity increases from A to $A' = \lambda A$ with $\lambda > 1$. Calculate the increase in consumption per capita, $\lambda_A = u'/u$.

3. Suppose that the population increases from L to $L' = \lambda L$ with $\lambda > 1$. Calculate the increase in consumption per capita, $\lambda_L = u'/u$.

4. Compare the values of λ_A and λ_L. Which one is larger, and what is the intuition for this result?

5. Explain why this model can generate growth in consumption per capita if the population grows over time.