Questions Regarding Econ 262A. 100 Points

1. (5 points) Let a General Mediation Model with a instrumental variable be defined by seven random variables in probability space $(\Omega, \mathscr{F}, \mathbf{P})$:

	Variable Description	**Model Equations**
1	Pre-program Variables:	$X = f_X(\epsilon_X)$
2	Instrumental Variable:	$Z = f_Z(X, \epsilon_Z)$
3	Unobserved Pre-treatment Counfounder:	$V = f_V(X, \epsilon_V)$
4	Treatment Choice:	$T = f_T(X, Z, V)$
5	Observed Mediator:	$M = f_M(X, T, V, \epsilon_M)$
6	Unobserved Mediator:	$U = f_U(X, T, V, T, \epsilon_U)$
7	Observed Final Outcome:	$Y = f_Y(X, T, M, U, V, \epsilon_Y)$

Express the general mediation model as a DAG.

2. (5 points) Suppose a researcher is interested in the identification of treatment effects of T, M on Y. The general mediation model can be simplified to key random variables that suffice to investigate the identification of treatment effects. This simpler model is often called the *Marginalized* or *Simplified* version of the mediation model. Write the equations of the simplified mediation model and draw its DAG representation.

3. (10 points) The assumption of Sequential Ignorability is often evoked to identify causal effects in the simplified mediation model. The assumption is stated using the language of counterfactual outcomes an consists of two conditional independence relations:

$$\Big(Y(t', m), M(t)\Big) \perp\!\!\!\perp T|K, \tag{1}$$

$$Y(t', m) \perp\!\!\!\perp M(t)|(T, K), \tag{2}$$

where K stands for some model variable. Which variable in the simplified mediation model plays the role of the matching variable K. Interpret the Sequential Ignorability (1)–(2) assumption. What is needed for (1)–(2) to hold?

4. (5 points) Consider the Generalized Roy Model with binary treatment defined by seven random variables in probability space $(\Omega, \mathscr{F}, \mathbf{P})$:

	Variable Description	**Model Equations**
1	Pre-program Variables:	$X = f_X(\epsilon_X)$
2	Instrumental Variable:	$Z = f_Z(X, \epsilon_Z)$
3	Unobserved Pre-treatment Counfounder:	$V = f_V(X, \epsilon_V)$
4	Treatment Choice:	$T = f_T(X, Z, V)$ where $T \in \text{supp}(T) \equiv \{t_1, t_0\}$
5	Unobserved Post-treatment Counfounder:	$U = f_U(X, T, V, \epsilon_V)$
6	Observed Final Outcome:	$Y = f_Y(X, T, V, U, \epsilon_Y)$
7	Counterfactual Outcomes:	$Y(t_1) = f_Y(X, t_1, V, U(t_1), \epsilon_Y)$
		$Y(t_0) = f_Y(X, t_0, V, U(t_0), \epsilon_Y)$

Draft the DAG of the Generalized Roy Model. Which of the statistical relations that hold among the variables $X, Z, V, T, U, Y, Y(t_1), Y(t_0)$ can be interpreted as a matching assumption?

5. (5 points) Display the DAG associated with the (marginalized) version of the generalized Roy Model that suffices to study the identification of the causal effect of T on Y. Express the matching condition of the previous question using this simplified version of the Roy model. If V were observed, how would you use this matching condition to identify the expected value of counterfactual outcomes $Y(t)$?

6. (5 points) How would you relate the matching assumption of the previous question with the method of Randomized controlled trials?

7. (5 points) The instrumental variable Z is an exogenous variable as it is not caused by any of the variables in the simplified Roy Model. State the key unconditional independent property of the instrumental variable Z that allows to identify treatment effects (under additional assumptions such as linearity, monotonicity or separability). State the exclusion restriction implied by this unconditional independence condition.

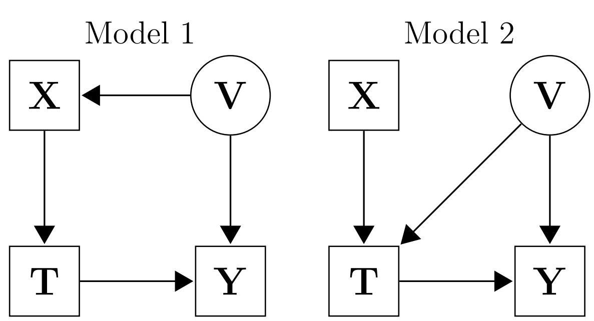
8. (5 points) Use $T_\omega(z)$ be the counterfactual choice of agent ω when the instrument Z is fixed at $z \in \text{supp}(Z)$ in the marginalized Roy Model of the previous question. Use this notation to state the monotonicity condition and its equivalent separability condition.

9. (5 points) Suppose that the instrument Z is continuous, then express the separability condition of the previous question in terms of the propensity score and a variable U that is a transformation of the unobserved variable V. Also state the distribution of the unobserved variable U.

10. (10 points) Let the marginal treatment effect of the binary Roy Model be given by $E(Y(t_1) - Y(t_0)|U = u)$, where U is the transformation of variable V discussed in the previous section. Use the reparability condition to show that the Average Treatment Effect $E(Y(t_1) - Y(t_0))$ can be expressed as a weighted average of the marginal treatment effect.

11. (10 points) Consider the simplified Roy Model with Binary Choice $T \in \{t_0, t_1\}$. Suppose that the instrumental variable Z in takes two values $\text{supp}(Z) \in \{z_0, z_1\}$. Let the response variable be $\mathbf{S} = [T(z_0)T(z_1)]'$. Suppose that the monotonicity condition $\mathbf{1}[T_\omega(z_0) = t_1] \leq \mathbf{1}[T_\omega(z_1) = t_1]$ holds. Describe the support of the response variable $\mathbf{S}$ and show how the 2SLS identifies the causal effect for compliers.

12. (10 points) Consider the models represented by two Directed Acyclic Graph (DAG) depicted below:



Variable V is an unobserved random variable in both models. Variables X, T, Y are observed continuous random variables. Suppose all structural equations associated with the DAGs above are linear. Describe a simple estimation procedure that identifies the causal effect of Y on T in both cases.

13. (20 points) Consider the models represented by two Directed Acyclic Graph (DAG) of the previous question. Suppose that T is binary and takes value in $\text{supp}(T) \in \{0,1\}$. Let $P(x) \equiv P(T = 1|X = x)$ be the propensity score of binary choice T. Describe two estimation procedures that use the propensity score $P(x)$ to identify the expected value of the counterfactual outcomes $E(Y(1))$ and $E(Y(0))$ for each model.