Department of Economics
UCLA

Spring 2010

Comprehensive Examination

Quantitative Methods

Answer all questions. Good luck!

Question 1:

Suppose that (X_1, X_2) possesses the density

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & x_1 > 0,\ x_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$.

(a) What is the joint density of (Y_1, Y_2)?

(b) Are Y_1 and Y_2 independent? Explain.

(c) What is the probability that $Y_1 \geq 1$ given that $Y_2 \leq .5$?

(d) What is the cumulative distribution function of Y_1?

(e) What is the Moment Generating Function of (Y_1, Y_2)?

Question 2:

Suppose that $X = (X_1, X_2)$,

$$Y_1^* = X_1\beta_1 + \varepsilon_1$$
$$Y_2^* = X_2\beta_2 + \varepsilon_2$$

and

$$W = \begin{cases} 1 & \text{if } Y_1^* \geq Y_2^* \\ 0 & \text{otherwise} \end{cases}$$

where $(\varepsilon_1, \varepsilon_2)$ is distributed independently of (X_1, X_2) with a Normal distribution, with mean $(0,0)$ and variances σ_{11} and σ_{22} and covariance σ_{12}. Suppose that W and X are observed, Y_1^*, Y_2^*, ε_1, and ε_2 are unobserved, and the support of (X_1, X_2) is R^K.

(a) Obtain an expression for the expectation of W given (X_1, X_2).

(b) Is $(\beta_1, \beta_2, \sigma_{11}, \sigma_{22}, \sigma_{12})$ identified? Explain.

(c) Suppose that $\sigma_{11} = 1$, $\sigma_{22} = 1$, and $\sigma_{12} = 0$. Is (β_1, β_2) identified? Explain.

(d) Suppose now that σ_{11} is a continuous function of X_1, $\sigma_{11}(x_1) = c(x_1) > 0$. Assume that $\sigma_{22} = 1$ and $\sigma_{12} = 0$. Are (β_1, β_2) and the function $c(\cdot)$ identified? If your answer is yes, prove it. If your answer is no, provide additional conditions under which they are identified and prove their identification under those conditions.

Question 3:

Suppose that

$$\begin{aligned} Y_1^* &= X_1\beta_1 + \varepsilon_1 \\ Y_2^* &= X_2\beta_2 + \varepsilon_2 \end{aligned}$$

and

$$Z = \begin{cases} Y_1^* & \text{if } Y_1^* \geq Y_2^* \\ Y_2^* & \text{otherwise} \end{cases}$$

Let $f_{\varepsilon_1,\varepsilon_2}$ denote the density of $(\varepsilon_1, \varepsilon_2)$.

(a) Obtain an expression for the expectation of Z given (X_1, X_2), in terms of $f_{\varepsilon_1,\varepsilon_2}$, X_1, X_2, β_1 and β_2.

(b) Obtain an expression for the density of Z given (X_1, X_2), in terms of $f_{\varepsilon_1,\varepsilon_2}$, X_1, X_2, β_1 and β_2.

Question 4:

You are given a following model:

$$\begin{aligned} y_i &= \alpha_1 + \varepsilon_i, \quad i = 1, \ldots, 50 \\ y_i &= \alpha_2 + \varepsilon_i, \quad i = 51, \ldots, 150 \end{aligned}$$

where ε_i $(i = 1, \ldots, 150)$ are iid $N(0,1)$. You are interested in $\alpha_2 - \alpha_1$.

In order to estimate $(\alpha_1, \alpha_2)'$, you generated the following variables

$$x_{i1} = 1, \quad i = 1, \ldots, 150$$
$$x_{i2} = \begin{cases} 0 & \text{if } i = 1, \ldots, 50 \\ 1 & \text{if } i = 51, \ldots, 150 \end{cases}$$

and considered the linear regression model:

$$y_i = x_{i1}\beta_1 + x_{i2}\beta_2 + \varepsilon_i, \quad i = 1, \ldots, 150$$

How would you estimate $\alpha_2 - \alpha_1$ in this model? What is the variance of your estimator? (You are expected to derive the exact variance, not the asymptotic variance.) Is your estimator normally distributed, or is it just asymptotically normal?

Question 5:

Consider the linear regression model $y_i = x_i\beta + \varepsilon_i$. We assume that (x_i, ε_i) are iid. We also assume that x_i and ε_i are independent of each other. Finally, we assume that $\varepsilon_i \sim N(0, 1)$. We consider two estimators of β:

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}, \quad \widehat{\beta}_2 = \frac{1}{n}\sum_{i=1}^n \frac{y_i}{x_i}.$$

Derive the asymptotic distributions of $\sqrt{n}\left(\widehat{\beta}_1 - \beta\right)$ and $\sqrt{n}\left(\widehat{\beta}_2 - \beta\right)$. Which one has the smaller asymptotic variance? Why? Hint (Jensen's Inequality): If φ is a convex function and if Z is a random variable, then $\varphi(E[Z]) \leq E[\varphi(Z)]$.

Question 6:

Consider the following two-equation model:

$$y_1 = \gamma_1 y_2 + \beta_{11}x_1 + \beta_{21}x_2 + \beta_{31}x_3 + \varepsilon_1$$
$$y_2 = \gamma_2 y_1 + \beta_{12}x_1 + \beta_{22}x_2 + \beta_{32}x_3 + \varepsilon_2$$

We assume that $E[\varepsilon_1 x_1] = E[\varepsilon_1 x_2] = E[\varepsilon_1 x_3] = E[\varepsilon_2 x_1] = E[\varepsilon_2 x_2] = E[\varepsilon_2 x_3] = 0$. Establish whether or not the following restrictions are sufficient to identify the model (or at least one of the equations):

(a) $\beta_{21} = \beta_{32} = 0$

(b) $\beta_{12} = \beta_{22} = 0$

(c) $\gamma_1 = 0$

(d) $\gamma_2 = \gamma_1$ and $\beta_{32} = 0$

Question 7:

Suppose that $X_1, \ldots, X_n$ are iid They have the distribution $N(\theta, 1)$.

(a) Discuss in detail the best test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

(b) Derive the maximum likelihood estimator $\widehat{\theta}$ for θ.

(c) Derive a consistent estimator of the asymptotic variance of $\sqrt{n}\left(\widehat{\theta} - \theta\right)$.

(d) Discuss in detail the LR test for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$. Here, the LR test denotes the asymptotic version of the test. You should derive the LR test statistic in detail. Reproducing the general formula will result in zero credit.

(e) Discuss in detail the Wald test for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$.

Question 8:

Consider a "random coefficient" binary discrete choice model where agent i's utility from alternative 1 is given by:

$$U_i = \theta_1 + (\theta_2 + v_i)\, w_i + \epsilon_i$$

where ϵ_i is a standard "logit error" (i.e. the difference between 2 iid Type 1 Extreme Value random variables with CDF $e^{-e^{-x}}$) and $v_i \sim N(0, \sigma^2)$. w_i is a scalar observed covariate. The unobservables ϵ_i and v_i are assumed independent of each other, independent across i, and independent of w_i. Agent i's utility from alternative 0 is normalized to 0.

(a) What are the location and scale normalizations in this model? Why are these normalizations necessary?

(b) Given a sample $i = 1, ..., N$, write down a smooth simulated likelihood function that can be maximized to obtain estimates of $(\theta_1, \theta_2, \sigma)$ (by "smooth", I mean that the likelihood function should be continuous in the parameter vector).

(c) Alternatively, write down a smooth simulated sample moment condition that can be minimized to obtain estimates of $(\theta_1, \theta_2, \sigma)$.

(d) Discuss the advantages and disadvantages of the estimator defined by 2) versus the estimator defined by 3).