UCLA Department of Economics

First Year Core Examination in
Quantitative Methods

Spring 2008

This is a 4 hour closed book/closed notes exam.

Answer ALL questions in Parts I, II, and III-
Use a separate answer book for each part.

Calculators and other electronic devices are not allowed.

GOOD LUCK!

Department of Economics
UCLA

Spring 2008

Quantitative Methods Comprehensive Examination

Directions: Please answer each of the three parts in a separate bluebook. You have four hours to complete the exam. Calculators and other electronic devices are not allowed.

Part I (based on Ec203A)

Question 1: The joint density of random variables X and Y is given by

$$f_{Y,X}(y,x) = \left\{ \begin{array}{lll} y & if & 0<y<1, \;\; 0<x<2 \\ & & \\ 0 & & otherwise \end{array} \right\}$$

(a) Are X and Y independently distributed? Explain.

(b) Calculate $\Pr(X+Y>2 \mid 0<Y<.5)$.

Question 2: Suppose that $X_1, ..., X_N$ are independent random variables, and each X_i is distributed binomial(m_i, p). That is, for $r = 0, ..., m_i$

$$\Pr(X_i = r) = \left(\begin{array}{c} m_i \\ r \end{array} \right) p^r \; (1-p)^{m_i - r}$$

(a) What is the moment generating function of X_i? Explain.

Let $Y = \sum_{i=1}^{N} X_i$.

(b) What is the distribution of Y? Explain.

(c) What is the expectation of Y? Explain.

(You may need to use the fact that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k \, b^{n-k}$.)

Question 3: Consider again the situation in Question 2, where $X_1, ..., X_N$ are independent random variables, but assume now that each X_i is distributed binomial(m, p). In other words, assume that $m_i = m$ for all i.

(a) Describe how to use the observations $X_1, ..., X_N$ to derive a Maximum Likelihood Estimator, $\widehat{p}$, for p.

(b) What is the asymptotic distribution of $\sqrt{N}\,(\widehat{p} - p)$? Explain.

(c) What is the asymptotic distribution of $\sqrt{N}\,(\widehat{p}^2 - p^2)$? Explain.

(d) Describe how you could use $\widehat{p}$ to test the hypothesis $H_0 : p = .5$, versus the alternative $H_1 : p \neq .5$, using a significance level α.

Part II (based on Ec203B)

PROBLEM 1:

Write the log-likelihood for n i.i.d. observations on (x_i, y_i) from the generalized normal linear regression model,

$$y_i|x_i \sim N(x_i\beta, \exp(x_i\gamma))$$

where β and γ are unknown parameters.

(i) Derive the ML estimator of β. Is it unbiased? Is it efficient? Show that it is consistent and derive its asymptotic distribution. (Hint: The estimator cannot be solved for explicitly)

(ii) Describe the GLS estimator of β (assuming γ is known). Is is unbiased? Is it efficient? Describe the conditions under whcih it is consistent and derive its asymptotic distribution. Which estimator would you prefer in large samples and why?

(iii) Describe the feasible GLS estimator when γ is unknown. Is it unbiased? Is it efficient? Discuss its consistency and its asymptotic distribution.

(iv) Describe the likelihood ratio, Wald and Lagrange Multiplier test for heteroskedasticity.

PROBLEM 2:

Suppose that your true model is

$$y_i = x_i\beta + \varepsilon_i$$

but you estimate the model

$$y_i = x_i\beta + z_i\gamma + v_i$$

where all you assume is that ε_i and v_i are uncorrelated with x_i and z_i. Derive the asymtotic properties of the OLS estimators of β in the true and the estimated model under random sampling. You may assume that all relevant moments exist.

Part III (based on Ec203C):

1. Consider the binary choice model

$$y_i^* = x_i'\beta_0 + \varepsilon_i,$$

where $\varepsilon_i|x_i \sim N(0, \sigma_\varepsilon^2)$, for $i = 1, ..., n$. Let

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0, \\ 0 & \text{Otherwise.} \end{cases}$$

Let

$$\begin{aligned} x_i' &= (1, x_{2i}, ..., x_{Ki}), \\ \beta_0' &= (1, \beta_2, ..., \beta_K) \end{aligned}$$

(a) Compute $\Pr(y_i = 1|x_i)$.

(b) Determine which of the model's parameters are identified. Justify your answer.

(c) Obtain the maximum likelihood estimator for β_0, say $\widehat{\beta}_n$.

(d) Let $l(\beta)$ denote the log-likelihood function for β. Show that

$$E\left[\frac{\partial l(\beta)}{\partial \beta}\frac{\partial l(\beta)}{\partial \beta'}\right] = -E\left[\frac{\partial^2 l(\beta)}{\partial \beta \partial \beta'}\right].$$

(e) Provide the asymptotic distribution for $\widehat{\beta}_n$ using the property estiblished in (d).

(f) Provide an estimate for the average marginal effect of x_{2i} on the probability that $y_i = 1$, conditional on x_i. Provide an estimate for the standard error of this marginal effect.

2. Consider the neo-classical regression model

$$y_i = x_i'\beta_0 + w_i^*\gamma_0 + u_i \qquad (i = 1, \ldots, n)$$

where β_0 is a $k \times 1$ vector of parameters, and γ_0 is a $p \times 1$ vector of parameters. Suppose that

$$\begin{aligned} &E[x_i u_i] = 0, \\ &w_i = w_i^* + v_i, \end{aligned}$$

with

$$E[v_i u_i] = 0.$$