Department of Economics **Spring 2007**
UCLA

Quantitative Methods Comprehensive Examination

Please answer each of the three parts in a separate bluebook. You have four hours to complete the exam. Calculators and other electronic devices are not allowed.

Part I (based on Ec203A)

Question 1: Let X have pdf $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, with support $-\infty \leq x \leq \infty$. Find the support and the pdf of $Y = X^2$.

Question 2: Let $\widehat{\theta}_n$ be an estimator of θ. Show that, if (1) $\lim_{n\to\infty} Var(\widehat{\theta}_n) = 0$ and (2) $\lim_{n\to\infty} Bias(\widehat{\theta}_n) = 0$ then $\widehat{\theta}_n$ is a consistent estimator of θ (i.e., $\lim_{n\to\infty} P(|\widehat{\theta}_n - \theta| \geq \varepsilon) = 0$). (Hint: use Chebyshev's inequality: $P(g(X) \geq r) \leq \frac{Eg(X)}{r}$).

Question 3: Let $X_1, ..., X_n$ be iid Bernoulli(p). Find the MLE of p. (Recall that, if $X \sim Bernoulli(p)$, the pdf is $f(x) = p^x(1-p)^{1-x}$).

Question 4: Let $X_1, ..., X_n$ be iid $N(\theta, \sigma^2)$, σ^2 known. Use the Neyman-Pearson lemma to find the uniformly most powerful test for $H_0 : \theta = 1$ against $H_1 : \theta = 0$, for a significance level $\alpha = 0.05$.

Part II (based on Ec203B)

Question 1: Consider the linear regression model given by

$$y_i = x_i\beta_0 + \varepsilon_i$$

where $E(\varepsilon_i|x_i) = 0$ and $E(\varepsilon_i^2|x_i) = \gamma_0 x_i^2$ where x_i is a scalar random variable and γ_0 is an unknown parameter. Make any additional assumptions you deem necessary to prove your claims.

(a) Consider the following statements: (a) The OLS estimator is unbiased. (b) The OLS estimator is consistent. (c) The OLS estimator is efficient. Are they right or wrong? Prove your claims.

(b) Derive the asymptotic distribution of the OLS estimator and propose two consistent estimators of its asymptotic variance. Prove your claims.

(c) Consider the WLS estimator of β_0. Is it unbiased? Is it consistent? Derive its asymptotic distribution and propose a consistent estimator of its asymptotic variance. Prove your claims.

Question 2:

For each one of the following claims show whether they are true or false.

(a) The OLS residuals are uncorrelated with the predicted values in the classical linear regression model.

(b) The R^2 of a k-variate regression does not change if we add to the dependent variable a constant and/or if we multiply the dependent variable by a constant.

(c) For the k-variate regression model, $y = X\beta + \varepsilon$, the fit as measured by R^2 does not change if we transform the X matrix by post-multiplying it by a $k \times k$ non-singular matrix.

(d) The Wald test statistic and the F statistic for testing a set of p linear restrictions on the coefficients of a $K-$ variate normal linear regression model coincide.

Part III (based on Ec203C)

Question 1: True/Questionable/False?
(i) When instruments are weak, an applied researcher should use OLS because the 2SLS estimator can be extremely biased.
(ii) In a linear endogenous regression model the asymptotic variance covariance matrix of the 2SLS estimator does not depend on the number of instruments used as long as the model is identified.
(iii) In a linear regression model with iid data, HAC estimation is consistent, only if the bandwidth S_T grows to infinity when $T \to \infty$.
(iv) When instruments are weak, an applied researcher doing inference based on inverting a Wald statistic may obtain misleadingly narrow confidence intervals.
(v) When testing overidentifying restrictions, a large value of the J-test is not necessarily evidence against the null hypothesis of instrument exogeneity due to the inconsistency of the test against certain alternatives.
(vi) With an MSE loss function, the optimal linear forecast of Y_t based on $(Y_{t-1}, ..., Y_1)$ does not depend on third and higher moments of the data *only if* the data is Gaussian.

Question 2: Take the linear model

$$\begin{aligned} y_i &= x_i\beta + e_i, \\ E(e_i|x_i) &= 0, \end{aligned}$$

where x_i and β are 1x1.
(a) Show that $E(x_i e_i) = 0$ and $E(x_i^2 e_i) = 0$.
(b) Is $z_i = (x_i, x_i^2)$ a valid instrumental variable for estimation of β?
(c) Define the 2SLS estimator of β, using z_i as an instrument for x_i. How does this differ from OLS?
(d) Find the efficient GMM estimator of β based on the moment condition

$$E(z_i(y_i - x_i\beta)) = 0.$$

Does this differ from 2SLS and/or OLS?

Question 3: Suppose that an MA(2) model is estimated by conditional MLE when the second moving average parameter, θ_2, is actually equal to zero. Derive an expression for the relative efficiency of the resulting estimator of θ_1 as compared with the estimator obtained from an MA(1) model.