Spring 2004 UCLA Department of Economics
Written Qualifying Examination in Quantitative Methods

Instructions:
Answer **ALL questions** in Parts I, II, and III
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

1 Part I

Question 1:

Suppose that

$$X \equiv \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right)$$

Let

$$U = X_2 - \frac{\rho\sigma_2}{\sigma_1} X_1$$

Show that U is independent of X_1.

Question 2:

Let $X_1, \ldots, X_n$ be i.i.d. $N(0, \theta)$. We would like to test $H_0 : \theta = \theta'$ against $H_1 : \theta < \theta'$, where θ' is a fixed positive number. Prove that the set $\{(x_1, \ldots, x_n) : \sum_{i=1}^n x_i^2 \leq c\}$ for some $c > 0$ characterizes a uniformly most powerful critical region.

Question 3:

Let $X_1, \ldots, X_n$ be i.i.d. with common PDF $f(x) = \theta^x e^{-\theta}/x!$, $x = 0, 1, 2, \ldots$ What is the MLE of $\Pr[X_{10} = 1 \text{ or } 2]$? Justify your answer.

Question 4:

Let $X_1, \ldots, X_n$ be i.i.d. $\sim$ Binomial $(1, \pi)$. Show that $\overline{X} = n^{-1} \sum_{i=1}^n X_i$ has the minimal variance among all unbiased estimators. Justify your answer by explicitly invoking some theorem(s).

2 Part II

Question 1:

Suppose that you have n i.i.d. observations on (Y_i, X_i) from the model:

$$\begin{aligned} Y_i &= X_i\beta_0 + \varepsilon_i \\ \varepsilon_i|X &\sim N\left(0, \sigma^2 \exp\left(Z_i\gamma_0\right)\right) \end{aligned}$$

where β_0 and γ_0 are unknown parameter vectors, σ^2 is an unknown positive constant and Z_i is a subvector of X_i.

1. Define and justify the maximum likelihood estimator of β_0.
2. Define and justify the feasible generalized least squares estimator of β_0
3. Evaluate the claim: "$\hat{\beta}_{ML}$ and $\hat{\beta}_{FGLS}$ are numerically identical".
4. Evaluate the claim: "$\hat{\beta}_{ML}$ and $\hat{\beta}_{FGLS}$ are asymptotically equivalent".

In parts (1) and (2) 'justify' means 'explain why one would use this estimator'. In parts (3) and (4) 'evaluate' means 'argue whether it is always true, or true under some conditions, or false'. Make sure to provide proofs for all your claims.

Question 2:

Suppose that we have an i.i.d. sample of n observations on (Y_i, X_i) from the censored regression model

$$Y_i = \max\left(Y_i^*, 0\right)$$

where the latent variable Y_i^* is given by a linear regression model

$$Y_i^* = X_i\beta_0 + \sigma_0\varepsilon_i$$

where $\sigma_0 > 0$ is unknown and ε_i is distributed according to a known continuous distribution F with density function f independent of X_i.

1. Define the maximum likelihood estimator of β_0.
2. Derive the conditional mean of Y_i given X_i, i.e. $E\left(Y_i|X_i\right)$.
3. Derive the marginal effect of X_i on $E\left(Y_i|X_i\right)$.
4. Evaluate the claim: "Censoring leads to an attenuation of the marginal effect of X_i relative to its effect in the latent regression."

Note that F need NOT be the cdf of the standard normal distribution.

3 Part III

Question 1: True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

1. A large value of the J statistic is evidence against the validity of the moment restrictions.
2. The process X_t that satisfies $X_t + 1.9X_{t-1} + .88X_{t-2} = \varepsilon_t + .2\varepsilon_{t-1} + .7\varepsilon_{t-2}$ for white noise ε_t is causal and invertible.
3. An MA(q) process (for a finite positive integer q) is ergodic.
4. If $X_n = O_p(n^{-\delta})$ for some $\delta > 0$ then $X_n = o_p(1)$.
5. When instruments are weak, an applied researcher should not use a Wald statistic to test parameter hypotheses but instead use the statistic LM_{CUE}.

Question 2: Consider the linear simultaneous equations model

$$\begin{aligned} Y &= X\beta + \varepsilon, \\ X &= Z\Pi + v, \end{aligned}$$

where Y is $n \times 1$, X is $n \times d$. The observations are i.i.d. and you can assume conditional homoskedasticity. The sample size n is larger than d. Interest focuses on estimation of β.

1. If Z is an $n \times n$ invertible matrix, show that 2SLS and OLS are identical.

   Now consider 2SLS estimators $\hat{\beta}_i$ $(i = 1, 2)$ based on instruments Z_i of dimensions $n \times k_i$, where $k_1 < k_2$ and Z_1 consists of the first k_1 columns of Z_2.
2. Verify that the variance of $\hat{\beta}_2$ is smaller than the one of $\hat{\beta}_1$ (in the positive definite sense).
3. Discuss the statement "The more instruments we use for 2SLS, the better the estimator becomes".

Question 3: Show that for a covariance stationary process Y_t, the linear projection of Y_{t+1} on a constant and Y_t is given by

$$\widehat{E}(Y_{t+1}|Y_t) = (1 - \rho_1)\mu + \rho_1 Y_t,$$

where $\mu := E(Y_t)$, $\rho_1 := \gamma_1/\gamma_0$, and γ_k, as usually, denotes the covariance of Y_t at lag k. Show that for an AR(2) process with AR parameters ϕ_i $(i = 1, 2)$, the implied forecast is

$$\mu + [\phi_1/(1 - \phi_2)](Y_t - \mu).$$