Department of Economics
UCLA

Spring 2003

Quantitative Methods Comprehensive Examination

This is a four hour closed-book examination. There are three parts in this exam. Please answer **ALL** parts of the exam. Use separate exam book for each of the three sections.

Calculators, or any other electronic devices, are not allowed.

Part I.

1. Suppose that $X \sim N(\theta_1, \theta_2)$. Let $\theta \equiv (\theta_1, \theta_2)'$. Compute the Fisher Information for θ.

2. Let $X_1, \ldots, X_n$ be i.i.d. with the following PDFs. In each case, find the asymptotic variance of $\sqrt{n}\left(\widehat{\theta}_{MLE} - \theta\right)$

   (a) $f(x;\theta) = \theta x^{\theta-1}$ for $0 < x < 1$ and zero elsewhere

   (b) $f(x;\theta) = (1/\theta)\exp(-x/\theta)$ for $0 < x$ and zero elsewhere

3. Let $X_1, \ldots, X_{25}$ be i.i.d. $N(\mu, 1)$. We wish to test

$$\begin{aligned} H_0 &: \mu = \mu_0 \\ H_1 &: \mu = \mu_1 \end{aligned}$$

   for some $\mu_1 > \mu_0$. Derive the best test, i.e., the best critical region, at the 0.05 level.

4. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* random variables such that $X_i \sim N(\mu, 1)$. We would like to test $H_0 : \mu = 0$ against $H_1 : \mu > 0$. A friend of yours suggested a testing strategy where the null hypothesis is rejected if and only if

$$S \equiv \frac{1}{n}\sum_{i=1}^{n} 1(X_i \geq 0) - \frac{1}{2} \geq \frac{1.96}{\sqrt{n}} \times \frac{1}{2}.$$

   Here, $1(\cdot)$ is an indicator function such that

$$1(X_i \geq 0) \equiv \begin{cases} 1 & \text{if } X_i \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

   What is the exact probability of Type I error of this test when $n = 4$? What is the limit of the probability of Type I error as $n \to \infty$?

Part II.

1. Consider a heteroskedastic linear regression model

$$y_i = x_i\beta + \varepsilon_i$$

where $\{\varepsilon_i\}_{i=1}^n$ are independent and distributed as $N(0, z_i\alpha)$, β is a vector of unknown parameters, z_i is a vector of known constants and α is a vector of unknown parameters. Assume that all the appropriate assumptions hold so that the OLS estimator of β is consistent and asymptotically normal.
Propose an estimator of α, show that it is consistent and derive its asymptotic distribution. State all necessary assumptions and theorems.

2. Consider the simple linear regression model

$$y_i = a + x_i\beta + \varepsilon_i,$$

where $\{\varepsilon_i\}_{i=1}^n$ are independent and distributed as $N(0, \sigma^2)$, and α and β are scalar unknown parameters.
Show that the three familiar tests, Wald, Likelihood Ratio, and Lagrange Multiplier tests, for testing the hypothesis $\beta = 0$ take the form:

$$\begin{aligned} W &= \frac{nr^2}{(1-r^2)} \\ LR &= n \ln\left(\frac{1}{1-r^2}\right) \\ LM &= nr^2 \end{aligned}$$

where r is the simple correlation coefficient between x and y.

Part III.

1. Consider the Neo Classical regression model

$$y_i = \beta' x_i + \gamma' w_i + u_i \qquad (i = 1, \ldots, n)$$

where β is a $k \times 1$ vector of parameters, and γ is a $p \times 1$ vector of parameters. Also, for x_i we have

$$E[x_i u_i] = 0$$

and for w_i we have

$$E[w_i u_i] \neq 0.$$

   (a) Can the coefficient vector β be consistently estimated by a least-squares regression? Demonstrate your answer as precisely as possible.

   (b) Suppose that $\mathrm{Cov}(x_i, w_i') = 0$, and that $X'W = 0$, where $X = (x_1, \ldots, x_n)'$, and $W = (w_1, \ldots, w_n)'$. Suppose also that the vector $z_i' = (z_{1i}, \ldots, z_{li})$ (with $l > p$) is a proper instrument for $w_i' = (w_{1i}, \ldots, w_{pi})$, and let $Z = (z_1, \ldots, z_n)'$. None of the elements in z_i equal any of the elements in x_i. Compute the instrumental variable estimator for γ in a regression that includes both x and w.

   (c) Under the condition in (b), consider the following estimation procedure: (i) Estimate β from a regression of y on X; and (ii) Compute $\hat{y} = My$ (where $M = I - X(X'X)^{-1}X'$) and estimate γ by computing the instrumental variable estimator from a regression of $\hat{y}$ on w, using z as the instrumental variable for w.

   (d) Compare the estimators for γ from (b) and (c3). Explain the difference and/or the similarity.

2. Consider the binary choice model

$$y_i^* = x_i' \beta_0 + \varepsilon_i,$$

for $i = 1, ..., n$, where $\varepsilon_i | x_i \sim$i.i.d. $G(\cdot)$, $G(\cdot)$ is independent of x_i and symmetric around zero.

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0, \\ 0 & \text{Otherwise.} \end{cases}$$

   (a) Compute $\Pr(y_i = 1 | x_i)$.

   (b) Demonstrate how to obtain the maximum likelihood estimator for β_0, say $\widehat{\beta}_n$.

   (c) Let $l(\beta)$ denote the log-likelihood function for β. Show that

$$E\left[\frac{\partial l(\beta_0)}{\partial \beta} \frac{\partial l(\beta_0)}{\partial \beta'}\right] = -E\left[\frac{\partial^2 l(\beta_0)}{\partial \beta \partial \beta'}\right].$$

   (d) Provide the asymptotic distribution for $\widehat{\beta}_n$ using the property established in (c).

   (e) Show how to test whether or not the marginal effect of x_{2i} on the probability that $y_i = 1$, conditional on x_i is of any significance. Justify your answer