UCLA Department of Economics

First Year Core Examination in
Quantitative Methods

Fall 2008

This is a 4 hour closed book/closed notes exam.

Answer ALL questions in Parts I, II, and III-
Use a separate answer book for each part.

Calculators and other electronic devices are not allowed.

GOOD LUCK!

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Spring 2008

Quantitative Methods Comprehensive Examination

Part I (based on Ec203A)

Question 1:

Let $X_1, ..., X_n$ be *i.i.d.* random variables, each distributed $U(\theta_1, \theta_2)$, where $\theta_1 < \theta_2$. In other words, for each i, the density, f_{X_i}, of X_i is

$$f_{X_i}(x) = \left\{ \begin{array}{ll} \frac{1}{\theta_2 - \theta_1} & \theta_1 \leq x \leq \theta_2 \\ & \\ 0 & otherwise \end{array} \right\}$$

a. Show that the mean, μ, and variance, σ^2, of X_i, are given by $\mu = (\theta_1 + \theta_2)/2$ and $\sigma^2 = (\theta_2 - \theta_1)^2/12$.

b. Provide consistent estimators for μ and σ^2. Prove that they are consistent.

c. Use your estimators for μ and σ^2 to derive consistent estimators, $\widehat{\theta}_1$ and $\widehat{\theta}_2$, for θ_1 and θ_2. Prove that your estimators are consistent.

d. Let

$$\widehat{\beta} = \sqrt{3n}\left(\widehat{\theta}_2 + \widehat{\theta}_1\right) / \left(\widehat{\theta}_2 - \widehat{\theta}_1\right)$$

where $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are the estimators that you derived in (c). Suppose, in this subquestion only, that $E(X_1) = 0$. Derive the limiting distribution of $\widehat{\beta}$. Justify your answer.

e. Develop a test for the hypothesis $H_0 : \theta_1 + \theta_2 = 5$ versus the alternative $H_1 : \theta_1 + \theta_2 < 5$. Justify your steps.

f. Assume now that $E(X_1) > 0$. Develop an approximate 95% confidence interval for the value of the parameter $\pi = \sqrt{(\theta_1 + \theta_2)}$.

Question 2:

Answer TRUE, FALSE, or UNCERTAIN, and justify.

1. Suppose that X and Y are independent random variables. Let $Z = g(X, Y)$ where g is continuous function. Then, Z and X are independently distributed.

2. Suppose that X and Y are random variables such that for some function h, $Y = h(X)$. If Y is continuous, X may be either discrete or continuous, but, if Y is discrete, X must be discrete.

3. Suppose that the distribution of X, conditional on Z, is normal with mean $2\,Z$ and variance σ^2, and that Z is normal with mean 0 and variance ω^2. Then, the unconditional distribution of X is normal with mean zero and variance $\sigma^2 + \omega^2$.

4. Suppose that the random vector (X, Z) and the random variable Y are independently distributed. Then, for all x, y, z, the conditional joint density of (X, Y) given Z and the conditional densities of X given Z and Y given Z, satisfy

$$f_{X,Y|Z=z}(x, y) = f_{X|Z=z}(x)\; f_{Y|Z=z}(y)$$

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Part II (based on Ec203B)

PROBLEM 1:

TRUE, FALSE. EXPLAIN

(a) Suppose that the true model is $Y_i = X_i\beta + \varepsilon_i$ where X_i is a scalar explanatory variable independent of ε_i. Instead you run the OLS regression $X_i = Y_i\delta + u_i$ and use $1/\hat{\delta}$ as an estimator of β. The proposed estimator is consistent for β.

(b) The R^2 from an OLS regression is the square of the simple correlation between the regressand and the predicted value.

PROBLEM 2:

Suppose that $y_i = x_i\beta + \varepsilon_i$, where $\varepsilon_i = \exp(z_i\delta)\, v_i$ and x_i is a sequence of i.i.d. random vectors, v_i is a sequence of i.i.d. random variables independent of x_i with $E(v_i) = 0$ and $V(v_i) = 1$, and z_i is a sequence of i.i.d. random vectors independently distributed of (x_i, v_i). Assume that the $k-$dimensional random vector x_i does not include a constant.

(a) Prove the consistency of the OLS estimator of β and derive the asymptotic distribution of $\sqrt{n}\left(\hat{\beta}_{OLS} - \beta\right)$.

(b) Suggest a course of action for obtaining the "best" estimate for β under the assumptions of the model. In what sense is it best?

PROBLEM 3:

Suppose you have n independent observations $\{(y_i, x_i)\}_{i=1}^{n}$, where y_i and x_i are scalar random variables. The density of y_i conditional on x_i is Gamma:

$$\frac{(\beta + x_i)^{-\rho}}{\Gamma(\rho)} y_i^{\rho-1} e^{-y_i/(\beta+x_i)}$$

Recall that for a Gamma distribution with density

$$\frac{\lambda^{\rho}}{\Gamma(\rho)} y^{\rho-1} e^{-\lambda y}$$

the mean is ρ/λ and the variance is ρ/λ^2. Propose three estimators for β and ρ, and discuss their asymptotic properties.

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Part III (based on Ec203C)

1. Consider the Seemingly Unrelated Regression (SUR) model

$$y_{ij} = x_{ij}'\beta_j + u_{ij} \qquad (i = 1, \ldots, n;\ j = 1, ..., J)$$

   (a) State the conditions for u_{ij} that would allow one to estimate the parameter vectors β_j, $j = 1, ..., J$, from J separate least-squares (LS) regressions.

   (b) Suppose now that for $u_i = (u_{i1}, ..., u_{iJ})'$ we have

$$u_i | x_{1i}, x_{2i}, x_{3i} \sim \text{i.i.d.}\,(0, \Sigma)\,.$$

   How would you obtain a feasible generalized least-squares (FGLS) estimator for $\beta_1, ..., \beta_J$. Justify your answers at each and every stage.

   (c) Suppose that it is given that $\beta_2 = \beta_3 = \beta$. How would you get efficient estimates for β and β_1? Justify your answers at each and every stage.

2. You are presented with a moment equations given by

$$\varphi(w, \beta),$$

where β is a $K \times 1$ vector of unknown parameters, and with data w_i, for $i = 1, ..., n$, where w_i is a $J \times 1$ vector of data. Also, $\varphi(w, \beta)$ is an $M \times 1$ vector-valued function. The true parameter vector is β_0, and our goal is to estimate β_0. Suppose that it has already been established that

$$E[\varphi(w, \beta_0)] \equiv E[\varphi(w, \beta)]|_{\beta=\beta_0} = 0.$$

(a) What are the minimal conditions required by the data and $\varphi(w, \beta)$ that would allow one to estimate β_0. Please justify all statements made.

(b) Suppose now that one suggested a different $M \times 1$ moment vector-valued function, say $\psi(w, \beta)$ that satisfies

$$E[\psi(w, \beta_0)] = 0,$$

with $M > K$. Describe in detail how to obtain the optimal generalize method of moments (GMM) estimator based on this latter function. Provide detailed justification.

(c) Suppose now that you are told that the model is given by

$$y_i = h(x_i, \beta_0) + \varepsilon_i.$$

What restrictions on ε_i would allow you to estimate β_0? Please justify your answer.

(d) Suppose the restrictions in (c) are satisfied. Suggest a moment function $\psi(w, \beta_0)$ as in (b), and justify your answer.