

Quantitative Methods Comprehensive Examination

Please answer each of the three parts in a separate bluebook. You have four hours to complete the exam. Calculators and other electronic devices are not allowed.

Part I (based on Ec203A)

Question 1:

Let X have a continuous and strictly increasing cdf $F(\cdot)$, and define $Y = F(X)$. Show that Y is uniformly distributed on $(0,1)$. (Hint: use the cdf of Y)

Question 2:

Let X have pdf $f(x) = e^{-x}$, for $x \geq 0$. Find the median and the mean of X .

Question 3:

Let X and Y be independent random variables, and let $g(x)$ be a function only of x and $h(y)$ be a function only of y . Show that $E[g(X) \cdot h(Y)] = E[g(X)] \cdot E[h(Y)]$

Question 4:

Let X_i , $i = 1, \dots, n$ be iid $N(\mu, \sigma^2)$, and consider the two estimators of σ^2 :

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \text{ and } \hat{\sigma} = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2.$$

Using the fact that $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$, show which of the two estimators has smaller Mean Squared Error. (Recall that, if Z is χ_c^2 , then $E(Z) = c$ and $Var(Z) = 2c$).

Part II (based on Ec203B)

Question 1:

Let y_i and z_i be scalar random variables and x_i a $1 \times K$ random vector. Consider a random sample of size n from the linear regression model given by

$$E(y_i|x_i, z_i) = x_i\beta_0 + \gamma_0 z_i$$

where $Var(y_i|x_i, z_i) = \sigma^2$ and $E(x_i'z_i) = 0$.

- (i) Evaluate the following estimators of β_0 in terms of bias, consistency and asymptotic efficiency: (a) The OLS estimator of β_0 from regressing y on both x and z , and (b) the OLS estimator of β_0 from regressing y on x alone.
- (ii) The coefficient of determination of a linear regression increases when we add a variable. (Assume that the model contains a constant term.). Is the previous statement true or false? Prove your claim.

Question 2:

Provide primitive conditions for consistency and asymptotic normality of the MLE, NLS and WNLS estimator of β_0 in the probit model:

$$\Pr(Y_i = 1|X_i) = \Phi(X_i\beta_0)$$

and derive their asymptotic distributions. Here Φ is the cdf of the standard normal distribution. Make sure to describe and justify all three estimators.

Part III (based on Ec203C)

Question 1: True/Questionable/False?

- (i) A large value of the J statistic is evidence against the validity of the moment restrictions.
- (ii) In a scalar location model with $Normal(0, \sigma^2)$ errors, the asymptotic variance of the sample median depends crucially on σ^2 .
- (iii) A stationary AR(1) process $y_t = \rho y_{t-1} + u_t$ (with u_t iid normal) is ergodic.
- (iv) If $X_n = O_p(n^{-\delta})$ for some $\delta > 0$ then $X_n = o_p(1)$.
- (v) When instruments are weak, an applied researcher should not use a Wald statistic to test parameter hypotheses but instead use the statistic LM_{CUE} .
- (vi) In a linear regression model with iid data, HAC estimation is consistent, only if the bandwidth S_T grows to infinity when $T \rightarrow \infty$.

Question 2: Consider the linear simultaneous equations model

$$\begin{aligned} Y &= X\beta + \varepsilon, \\ X &= Z\Pi + v, \end{aligned}$$

where Y is $n \times 1$, X is $n \times d$. The observations are i.i.d. and you can assume conditional homoskedasticity. The sample size n is larger than d . Interest focuses on estimation of β .

- i) If Z is an $n \times n$ invertible matrix, show that 2SLS and OLS are numerically identical. Now consider 2SLS estimators $\hat{\beta}_i$ ($i = 1, 2$) based on instruments Z_i of dimensions $n \times k_i$, where $k_1 < k_2$ and Z_1 consists of the first k_1 columns of Z_2 .
- ii) Verify that the asymptotic variance of $\hat{\beta}_2$ is smaller than the one of $\hat{\beta}_1$ (in the positive definite sense).
- iii) Discuss the statement “The more instruments we use for 2SLS, the better the estimator becomes”.

Question 3: i) For a covariance stationary process Y_t , derive the linear projection of Y_{t+1} on a constant and Y_t .

- ii) What is the implied forecast for an AR(2) process with AR parameters ϕ_i ($i = 1, 2$)?