# UCLA Department of Economics

## First Year Core Examination in
## Quantitative Methods

**Fall 2006**

**This is a 4 hour closed book/closed notes exam.**

**Answer ALL questions in Parts I, II, and III-**
**Use a separate answer book for each part.**

**Calculators and other electronic devices are not allowed.**

**GOOD LUCK!**

Department of Economics
UCLA Fall 2006

# Quantitative Methods Comprehensive Examination

Please answer each of the three parts in a separate bluebook. You have four hours to complete the exam. Calculators and other electronic devices are not allowed.

## Part I (based on Ec203A)

1. (10 pt.) Let $X_1, \ldots, X_n$ be a random sample from a distribution with pdf $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere. Prove that the best critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ takes the form
$$\left\{ (x_1, \ldots, x_n) : \prod_{i=1}^{n} x_i \geq c \right\}$$
You will get zero credit if you simply write that it is a consequence of Neyman-Pearson.

2. (10 pt.) Prove the following statement: Given a random vector $X$ with pdf $f(x; \theta)$, where $\theta$ is a scalar, we have
$$E\left[\left(\frac{\partial \log f(X; \theta)}{\partial \theta}\right)^2\right] = -E\left[\frac{\partial^2 \log f(X; \theta)}{\partial \theta^2}\right].$$
You will get zero credit if you simply say that it is a consequence of information equality. In other words, I expect you to prove the information equality.

3. (10 pt.) Suppose that $X_1, \ldots, X_n$ are *i.i.d.* We know that
$$E[X_i] = \alpha\beta, \quad \mathrm{Var}(X_i) = \alpha\beta^2$$
Construct method of moments estimator $\left(\widehat{\alpha}, \widehat{\beta}\right)$ for $(\alpha, \beta)$.

4. (10 pt.) Suppose that $X_i \sim N(\mu_i, \sigma_i^2)$ are independent of each other. Prove that $\sum_i a_i X_i \sim N\left(\sum_i a_i \mu_i, \sum_i a_i^2 \sigma_i^2\right)$.

5. (10 pt.) Suppose that $(\varepsilon_i, x_i, z_i)'$ are iid such that
$$\begin{pmatrix} \varepsilon_i \\ x_i \\ z_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 \end{bmatrix}\right)$$
Derive the asymptotic distribution of
$$\frac{n^{-1/2} \sum_{i=1}^{n} z_i \varepsilon_i}{n^{-1} \sum_{i=1}^{n} z_i x_i}$$

## Part II (based on Ec203B)

1. Define the logit model. Describe the three classical tests for testing a nonlinear hypothesis on the set of unknown coefficients for this particular model. Make sure to provide all details on the construction of the test statistics.

2. Consider the non-linear regression model given by

$$y_i = g\left(x_i \beta_0\right) + \varepsilon_i$$

In the following make any additional assumptions that you need to prove your claims.

   (a) First assume that $E\left(\varepsilon_i x_i\right) = 0$. Consider the following statements: (a) The NLS estimator is unbiased. (b) The NLS estimator is consistent. Are they right or wrong? Prove your claims.

   (b) Next assume that $\varepsilon_i$ and $x_i$ are stochastically independent. Consider the following statements: (a) The NLS estimator is unbiased. (b) The NLS estimator is consistent. Are they right or wrong? Prove your claims.

   (c) Derive the asymptotic distribution of the NLS estimator in the case(s) in parts (a) and (b) above where the estimator is consistent.

## Part III (based on Ec203C)

1. Consider the Seemingly Unrelated Regression (SUR) model

$$y_{ij} = x_{ij}'\beta_j + u_{ij} \qquad (i = 1, \ldots, n;\ j = 1, ..., J)$$

   (a) State the conditions for $u_{ij}$ that would allow one to estimate the parameter vectors $\beta_j$, $j = 1, ..., J$, from $J$ separate least-squares (LS) regression.

   (b) Suppose that $J = 3$. Also, assume that $x_{1i}$ and $x_{2i}$ are vectors of exogenous regressors, while for $x_{3i}$ we have $E\left[u_{3i}x_{3i}\right] \neq 0$. State the condition(s) under which one will be able to obtain a consistent estimator for $\beta_3$. Be as precise as possible in stating your assumption(s).

   (c) Suppose now that for $u_i = (u_{i1}, ..., u_{iJ})'$ we have

$$u_i | x_{1i}, x_{2i}, x_{3i} \sim \text{i.i.d.}\,(0, \Sigma)\,.$$

   How would you obtain a feasible generalized least-squares (FGLS) estimator for $\beta_1, ..., \beta_J$. Justify your answers at each and every stage.

2. Consider the non-linear model given by

$$y_i = g\left(x_i'\beta_0\right) + \varepsilon_i,$$

for $i = 1, ..., n$. Define the following moment equations:

$$\begin{aligned} \varphi_1\left(y_i, x_i, \beta\right) &= \left(y_i - g\left(x_i'\beta\right)\right) \frac{\partial g\left(x_i'\beta\right)}{\partial \beta} \\ \varphi_2\left(y_i, x_i, \beta\right) &= \left(y_i - g\left(x_i'\beta\right)\right) h\left(x_i\right), \end{aligned}$$

where $x_i$ is a $K \times 1$ vector and $h(\cdot)$ is a $P \times 1$ vector-valued function, with $P > K$.

(a) Suppose that $E\left[\varepsilon_i | x_i\right] = 0$. Show that solving for $\widehat{\beta}$ from

$$\sum_{i=1}^{n} \varphi_1\left(y_i, x_i, \beta\right) = 0$$

is equivalent to obtaining an estimate for $\beta$ from

$$\min_{\beta} \sum_{i=1}^{n} \left(y_i - g\left(x_i'\beta\right)\right)^2.$$

(b) Under the above conditions show that

$$E\left[\varphi_2\left(y_i, x_i, \beta_0\right)\right] = 0.$$

(c) Propose optimal GMM estimators for $\beta_0$ based on $\varphi_1\left(y_i, x_i, \beta\right)$ and $\varphi_2\left(y_i, x_i, \beta\right)$. Denote the estimators by $\widehat{\beta}_1$ and $\widehat{\beta}_2$, respectively.

(d) Provide the asymptotic properties for the estimators $\widehat{\beta}_2$ obtained in (c).

(e) Which of the two estimators would you prefer $\widehat{\beta}_1$ or $\widehat{\beta}_2$. Explain.

(f) Determine whether or not it is possible to obtain an estimator that is more efficient than the ones obtained in (c), based only on the moment functions defined above. Proved a detailed justification for your answer.