UCLA Economics

Fall 2001 Quantitative Methods Comprehensive Examination **4hours**

There are three sections with two questions in each section. Please answer all the questions; use a separate blue book for each of the three sections. For your information, the .90 and 0.95 quantiles of the Chi-squared distribution with one degree of freedom are 2.78 and 3.84.

Part I.

1. Let X be a binary variable with $Pr(X = 1) = 1/2$. Conditional on $X = x$, the random variable Y has a Poisson distribution with parameter $1 + (\theta - 1)x$.

   (a) What is the mean of Y.

   (b) What is the probability that $X = 1$ given $Y = 0$.

   (c) Suppose that $(X_1, Y_1), \ldots, (X_N, Y_N)$ are a random sample from this distribution. What is the maximum likelihood estimator for θ?

   (d) What is the large sample variance for the maximum likelihood estimator?

   (e) Construct a moment estimator for θ based on the the expected value of Y. What do you expect the variance of this estimator to be relative to the variance of the maximum likelihood estimator? There is no need to actually calculate this variance.

   Note: the probability function for a Poisson random variable Z with parameter λ is

$$f_Z(z; \lambda) = \frac{\lambda^z \exp(-\lambda)}{z!}.$$

2. Let the marginal distribution of X be binomial with parameters N and p.

   (a) What is the Cramer-Rao bound for unbiased estimators for p, given $N = 1000$?

   (b) Let $N = 1000$, and $X = 200$. Test the null hypothesis that $p = 0.16$ against the alternative that $p \neq 0.16$ at the 5% level using a likelihood ratio test.

   (c) Carry out the same test using a Lagrange multiplier or score test.

   (d) Carry out the same test using a Wald test.

(e) Suppose you want the likelihood ratio test of the null that $p = 0.16$ against the alternative that $p \neq 0.16$ to have power greater than or equal to 0.90 against the alternative $p = 0.18$. What is the minimum value of N you would need if the test is at the 5% level?

You can use the large sample approximation to the distribution of all the test statistics.

Part II.

1. Suppose that the Generalized Classical Normal Regression model applies to n observations from

$$y_i = x_i\beta + \varepsilon_i \qquad i = 1, ..., n$$

where the variance of ε_i is $\exp(w_i\alpha)$ for some observable non-stochastic $q-$dimensional variable w_i.

(a) Derive the GLS estimator of β assuming α is known.

(b) Describe the feasible GLS approach when α is unknown. Make sure to provide a consistent estimator of α and to justify it.

(c) State conditions under which the feasible GLS estimator is consistent and derive its asymptotic distribution.

2. Suppose that Classical Regression model holds for $y = \beta_0 + \beta_1 x + \beta_2 w + \varepsilon$ and you have obtained the OLS results: $\hat{\beta}_1 = 4$, $\hat{\beta}_2 = 0.2$, with estimated variances 2 and 0.06 and estimated covariance 0.05. You wish to test the hypothesis that β_1 is the inverse of β_2.

(a) Calculate the relevant test statistic justifying your approach in detail.

(b) Describe alternative testing procedures under the assumption of joint normality of the errors.

Part III.

1. Suppose that (ϵ_i, x_i, z_i), $i = 1, 2, \ldots$ is an i.i.d. sequence with

$$\begin{pmatrix} \epsilon_i \\ x_i \\ z_i \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & .5 \\ 0 & .5 & 3 \end{pmatrix}\right).$$

Let

$$y_i = x_i\beta + \epsilon_i.$$

(a) Show that the least squares coefficient in a regression of y on x is consistent for β, and derive its limiting distribution. (Be as precise as possible about its limiting distribution.)

(b) Next, consider the estimator

$$b_z = \sum_{i=1}^{n} z_i y_i / \sum_{i=1}^{n} z_i x_i.$$

Derive its plim and its asymptotic distribution. Again, be as precise as possible about the asymptotic distribution. Compare its asymptotic properties to the LS estimator in (a).

(c) Generalize the model, so that

$$\begin{pmatrix} \epsilon_i \\ x_i \\ z_i \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_{\epsilon\epsilon} & \sigma_{\epsilon x} & 0 \\ \sigma_{\epsilon x} & \sigma_{xx} & \sigma_{xz} \\ 0 & \sigma_{xz} & \sigma_{zz} \end{pmatrix}\right).$$

The model for y is the same as before, and you observe (y_i, x_i, z_i) for a random sample of size n. Provide a test of the hypothesis that $\sigma_{\epsilon x} = 0$, being as specific as possible about the limiting distribution of the test statistic.

2. Suppose that u_i is i.i.d. with cumulative distribution function

$$F(u) = e^u/(1 + e^u),$$

for $-\infty < u < \infty$.

(a) Show that the distribution of u is symmetric about 0.

(b) Let x_i be i.i.d., independent of u_i, and let

$$y_i^* = x_i'\beta + u_i.$$

You observe a sample of size n on x_i and

$$y_i \equiv \begin{cases} y_i^* & \text{if } y_i^* \leq 100 \\ 100 & \text{otherwise.} \end{cases}$$

Write the conditional log-likelihood function, providing as simple an expression as possible.

(c) What is the solution to the following problem: select a function $g(x)$ to minimize

$$E[(y_i - g(x_i))^2]$$

(Provide as complete an answer as possible.)