Macroeconomic/Monetary Economics
Ph.D. Field Examination
UCLA Department of Economics

September 2008

Instructions: You have 4 hours to complete the exam. Answer all three parts. Each part has equal weight.

Answer each part in a separate blue book.

Part 1

An economic model is represented by the following set of non-linear equations

$$y_t = E_t\left[\frac{x_t y_{t+1}}{a + y_{t+1}} \mid Z^t\right] + u_t, \tag{1}$$

$$x_t = c + b x_{t-1} + v_t, \tag{2}$$

$$x_0 = \bar{x}_0. \tag{3}$$

The notation is defined as follows. $y_t \in R$, $x_t \in R$ and u_t and v_t are scalar random variables with known distribution function D, where D has support $\bar{D} \subset R^2$. u_t and v_t are i.i.d., with zero means and covariance matrix equal to the identity matrix. $Z^t = \{x_s, y_s\}_{s=0}^{t}$, refers to the history of realizations of x_t and y_t up to and including date t. The parameters a, b and c are non-negative and $0 < b < 1$. $E_t\left[\cdot|Z^t\right]$ is the expectations operator conditional on Z^t.

1. What would be a suitable point around which to linearize this system of equations?

2. Write down a linear (not a log-linear) approximation to Equation (1). In what sense is this a good approximation? Define what is meant by a rational expectations equilibrium to this linear approximation. Is this equilibrium unique? is so why? If not why not?

3. Assume that the economist observes a finite sequence Z^T of draws from a model generated by equations (1)–(3). Describe how to construct an algorithm using the QZ decomposition that finds the unique solution to equations (1)–(3) (when it exists) in state space form for any given values of a, b and c.

4. Describe how to use the Kalman filter to construct the likelihood function $L\left(Z^T, \theta\right)$. You may assume at this point that u and v are jointly normal (even if this contradicts the assumptions required for the approximation to be good).

Part 2

Consider an economy in which human capital h can be put to three different uses: it can be used as h_W by workers for production of the final good, as h_P by professors who are engaged in producing new human capital, or as h_S by students who are also engaged in producing new human capital. At the aggregate level, the three uses of human capital have to add up to the economywide stock of human capital:

$$h_{W,t} + h_{P,t} + h_{S,t} = h_t.$$

The representative consumer has preferences described by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}.$$

The production function for the final good is given by:

$$y_t = A h_{W,t}.$$

The law of motion for human capital is:

$$h_{t+1} = B h_{S,t}^{\alpha} h_{P,t}^{\theta} X^{1-\alpha-\theta},$$

where $\alpha + \theta < 1$ and $X = 1$ is a factor in fixed supply owned by the households (such as land).

1. Formulate the social planning problem for this economy, and characterize the socially optimal allocation of human capital and the optimal long-run outcome.

2. Define a competitive equilibrium for this economy in which human capital is produced by firms like any other good (i.e., human capital can be purchased at some price in the market). Characterize the equilibrium price of human capital.

3. Define an equilibrium for this economy in which human capital is produced by private universities who pay their professors, charge tuition to their students, hand out the newly produced human capital to their students, and make zero profits. Characterize the equilibrium tuition.

4. Define an equilibrium for this economy in which human capital is produced by public universities who pay their professors, do not charge tuition, hand out the newly

produced human capital for free to their students, and have the possibility of limiting enrollment. The professors' salaries are financed via a lump-sum tax. Does an equilibrium exist if there is no enrollment limit?

5. Could the public university system ever achieve higher long-run output than the private systems? Could this ever be optimal?

Part 3

This part has *eight* questions divided amongst three sections. For full credit, you need to answer at least *three* of the nine questions. Extra credit will be given if additional questions are answered. You must attempt at least one question from section II, and one question from section III.

Your answers should be formally precise. Although no formal proofs are necessary, you should formally state all relevant results, and at least provide a sketch of the argument underlying the results.

Consider an economy with a measure one continuum of households. The households evaluate a sequence of consumption and labor supply choices allocations and labor supply choices $\{c_t, n_t\}_{t=0}^{\infty}$ according to the utility functional $U = \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t)]$.

Each period, households receive an idiosyncratic shock $\theta \in \{\theta_L, \theta_H\}$, which characterizes the efficiency of its labor supply; a household whose labor supply is n_t provides θn_t efficiency units of labor. θ evolves according to a first-order Markov process with transition probabilities $\pi(\theta'|\theta)$. The fraction of households who start in state θ_L at date 0 is λ_0.

In addition, there is a representative firm, which produces output of the consumption good c using *efficiency units of labor* y according to the technology $c = y$.

Questions $(i) - (v)$ below, consider a utilitarian social planner who seeks to maximize the agents' average expected utility, as of date 0.

Section I: (i) *Full commitment:* Characterize the optimal allocations, if the planner can fully enforce all transfers to and from households. Discuss a decentralization of this allocation in a competitive equilibrium.

Section II: (Answer at least one question from this section)

(ii) *Limited commitment:* Suppose now that all transfers from households to the planner must be voluntary. If a household defaults on a scheduled payment, the household no longer receives any transfers from the planner, but the household can still participate in the labor market, and consume his labor income. Characterize the resulting optimal allocations with limited commitment, and discuss if and how they may be decentralized in a competitive equilibrium.

(iii) *Private information:* Suppose now that the idiosyncratic labor supply shocks are *privately* observed by the household. Characterize optimal allocations with private information, and discuss if and how they may be decentralized in a competitive equilibrium. What are the resulting implications for long-run inequality in this economy?

(*iv*) *Moral hazard:* Suppose now that the transition probabilities are also a function of an unobserved effort choice $e > 0$ by the agent: $\pi(\theta'|\theta, e)$, where $\pi(\theta_H|\theta, e)$ is an increasing, concave function of e, for $\theta \in \{\theta_L, \theta_H\}$. Each period, there is a linear disutility of effort, so the household maximizes $\sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t) - e_t]$. Set up and discuss how you can characterize the solution to the optimal planning problem. What can you say about the solution to this problem?

Section III: (Answer at least one question from this section)

(*v*) *Limited commitment and private information:* Suppose now that labor supply shocks are privately observed by the households, and that there is limited commitment. How do you think this will alter the conclusions you have derived in the previous parts (*ii*) and/or (*iii*)?

(*vi*) *Incomplete markets:* Consider now a competitive version of this model, in which households trade a single, risk-free bond, in addition to the spot labor market. Set up and define the competitive equilibrium of this economy, and if possible, provide a sketch argument for the existence of equilibrium.

(*vii*) *Adding Capital:* Suppose now that the representative firm also used capital for production, according to a production function with constant returns to scale: $c = f(k, y)$, that capital depreciates at a constant rate δ, and that new investment has to be funded out of aggregate output. How would this affect your solution to the preceding planning problems?

(*viii*) *Quantitative Implications:* Some authors have argued that models along the lines of the ones sketched here can account for trends in income inequality, international capital flows, the equity premium, firm dynamics, or business cycles. Using one of these phenomena as the basis for your discussion, explain how you would formulate such a hypothesis within a model and confront it with the data.

Final suggestion: for some of the problems above, you may find it convenient to specialize the model. You may do so by assuming for example that this is a small open economy that can borrow/save at fixed world interest rate R, or by further specializing the process for idiosyncratic shocks, for instance by assuming that the process has an absorbing state. Obviously, it is always nice to have more general statements, but in some cases, you may find it useful to exploit the extra mileage that you can get out of such simplifications.