UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2016)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Equilibrium with Uncertainty

Consider a two-period exchange economy with one good and two consumers. There are two states of the world s_1 and s_2, which are equally likely. At state s_1, consumer 1 is endowed with 2 goods and consumer 2 is endowed with 1 good At s_2, consumer 2 is endowed with 2 goods and consumer 1 with 1 good. In the first period, a state of the world realizes and nothing else happens. Consumption occurs only in the 2nd period. Each consumer is an expected utility maximizer with a log (Bernoulli) utility function $u(x) = \log x$. Thus consumer i's expected utility from consumption plan $x_i = (x_{i,1}, x_{i,2})$ is given by $0.5 \log x_{i,1} + 0.5 \log x_{i,2}$ ($x_{i,s}$ is consumer i's consumption at state s). Answer the following questions.

(a) Find all the Pareto efficient allocations.

(b) Find an Arrow-Debreu equilibrium (remember that it is just a usual Walrasian equilibrium where goods are state-contingent goods).

Assume that the following two financial assets are available for trading in the first period for the rest of the questions. Asset A pays out 1 (unit of account) in both states in the 2nd period. Asset B pays 2 at state s_1 and 3 at state s_2 in the second period. Let q_k be the price of Asset k for $k = A, B$.

(c) Show that there is an opportunity for arbitrage when $(q_A, q_B) = (3, 5)$.

(d) Find asset prices (q_A, q_B) given which there is no arbitrage opportunity.

(e) Find a financial equilibrium/Radner equilibrium (consumption, asset holding, prices of the assets and state-contingent goods) that implements the same allocation as the Arrow-Debreu equilibrium allocation in (b).

2. Equilibrium with Indivisible Goods

We usually assume that goods are divisible: a consumer can consume any positive amount of any good. But what would happen if goods are *indivisible*? Many goods are indeed indivisible in real world. For example, you can buy 1 laptop or 2 laptops, but not 1.2 laptop. Here we consider a simple two good-two person pure exchange economy where goods are indivisible (Formally the set of feasible consumption vectors for consumer i is $X_i = \{(k_1, k_2) \,|k_1, k_2 \in \mathbb{N}\}$ and consumer i's endowment e_i is a pair of natural numbers). Assume that consumers' utility functions are linear and strongly increasing in both goods, i.e. $u_i(x) = \alpha_i x_{i,1} + \beta_i x_{i,2}$ with some $(\alpha_i, \beta_i) \gg 0$.

(a) Write down the conditions for $(x_1^*, x_2^*, p^*) \in X_1 \times X_2 \times \mathbb{R}_+^2$ to be a Walrasian equilibrium in this economy.

(b) Explain why every Pareto-efficient allocation must be on the boundary of the Edgeworth box when $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\beta_2}$ (For question (b)-(d), a graphical argument would suffice).

(c) Does there always exist a Walrasian equilibrium in this economy? (Hint: consider using a Pareto-efficient allocation).

(d) Show by an example that there may exist a Walrasian equilibrium in which the equilibrium allocation is not on the boundary of the Edgeworth box (hence is not Pareto-efficient by (b)).

(e) Suppose that good 1 is indivisible, but good 2 is divisible as usual. Does the first welfare theorem hold in this case? If you think so, provide a full proof. If not, find a counter example.

3. Repeated Games

ROW and COL play the following asymmetric version of Prisoner's Dilemma infinitely often. They discount future payoffs at the constant rate $\delta > 0$.

	C	D
C	$(4, 4)$	$(-2, 5)$
D	$(2, 0)$	$(1, 1)$

(a) Find the smallest discount factor for which there is a SGPE in which (C,C) is played every period.

(b) Find the smallest discount factor for which there is a SGPE in which play alternates (C,C), (D,C), (C,C), (D,C), ...

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4 Differentiated Commodities

Two firms produce differentiated commodities for sale in a single market. The firms have 0 fixed costs and constant marginal costs $c_1, c_2 \geq 0$. The market demands are

$$\begin{aligned} q_1 &= (1 - p_1 + 2p_2)^+ \\ q_2 &= (2 + p_1 - p_2)^+ \end{aligned}$$

Suppose first that the firms choose prices simultaneously so that the firms are playing a strategic form game.

(a) For what values of c_1, c_2 (if any) is there a Nash equilibrium in pure strategies in which both firms sell a positive quantity? For these values (if any), find (at least) one.

(b) For what values of c_1, c_2 (if any) is there a Nash equilibrium in pure strategies in which *only firm 1* sells a positive quantity? For these values (if any), find (at least) one.

(c) For what values of c_1, c_2 (if any) is there a Nash equilibrium in pure strategies in which *only firm 2* sells a positive quantity? For these values (if any), find (at least) one.

In all of the above, don't worry about knife-edge cases in which one firm is indifferent to operating or not.

Now suppose that firm 1 chooses its price first and firm 2 observes the choice of firm 1 before choosing its price, so that the firms are playing a sequential/extensive form game.

(d) For what values of c_1, c_2 (if any) is there a (pure strategy) subgame perfect equilibrium in which both firms sell a positive quantity? For these values (if any), find (at least) one.

(e) For what values of c_1, c_2 (if any) is there a (pure strategy) subgame perfect equilibrium in which *only firm 1* sells a positive quantity? For these values (if any), find (at least) one.

(f) For what values of c_1, c_2 (if any) is there a (pure strategy) subgame perfect equilibrium in which *only firm 2* sells a positive quantity? For these values (if any), find (at least) one.

In all of the above, don't worry about knife-edge cases in which one firm is indifferent to operating or not.

5. Efficient Mechanisms

The social surplus in an economy is $S(\theta, q) = \sum_{i=1}^{N} [B_i(\theta_i, q_i) - c_i q_i]$ for $q = (q_{1,\ldots,}q_I) \in Q$.

(a) Give this model both a private good interpretation and a public good interpretation.

(b) What is the marginal contribution to social surplus V-C-G mechanism for this model?

(c) For the remainder of this question, a single commodity can be produced at a cost $c \in [\alpha, \beta]$. Use the above V-C-G mechanism to design an efficient mechanism for allocating a single good to one of I buyers where buyer i has a value $\theta_i \in [\alpha, \beta]$ and a buyer's value is independently distributed with distribution function $F(\theta_i) \in \mathbb{C}^1$.

(d) Explain why there is no efficient mechanism for which the expected revenue of the designer is higher.

(e) Prove that this is true.

6. Indirect Price Discrimination

A type θ buyer's benefit from consuming q units is $B(\theta, q) = \theta q - \frac{1}{2}(4 - \theta) q^2$. The population mass is 1. The cost of production is $c \in (1, 2)$. Types are distributed on $\Theta = [1, 2]$ with distribution function $F(\theta) = \theta - 1$ Let $\{q(\theta), r(\theta)\}_{\theta \in \Theta}$ be an incentive compatible mechanism.

(a) Derive necessary conditions for incentive compatibility.

(b) Use them to show that the incentive compatible expected revenue of the designer is

$$\mathbb{E}\left[B(\theta, q) - \frac{1 - F(\theta)}{f(\theta)} \frac{\partial B(\theta, q)}{\partial \theta} - U(1)\right],$$

where $U(\theta)$ is type θ buyer's utility.

(c) Solve for the profit maximizing allocations $\{q(\theta)\}_{\theta \in \Theta}$.

(d) Explain why the profit maximizing outcome can be implemented as a non-linear pricing scheme in which consumers must pay $R(q)$ to purchase q units.

(e) Solve for the mapping $R(q)$ from number of units in the plan to the plan price.