UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2013)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Pareto Efficient Allocation and Social Welfare Maximization

Consider a private ownership economy $\mathcal{E}^{priv} = \left(\left\{\mathbb{R}^L_+, \succeq_i, e_i\right\}_{i=1,...,I}, \left\{Y_j\right\}_{j=1,...,J}, \left\{\theta_{i,j}\right\}_{i,j}\right)$ where $\succeq_i$ can be represented by a concave utility function u_i for each i and the total production set $Y = \sum_{j=1}^{J} Y_j$ is convex. Answer the following questions.

(a) Let U be the **utility possibility set** given by

$$U = \left\{u \in \mathbb{R}^I | \exists \left(x, y\right) \in A, u \leq u\left(x\right)\right\},$$

where A is the set of feasible allocations and $u\left(x\right) = \left(u_1\left(x_1\right), ..., u_I\left(x_I\right)\right)^\top$. Show that U is a convex set.

(b) Define **Pareto efficient allocation** in this economy. Show that, for any Pareto efficient allocation $\left(x^*, y^*\right)$, $u\left(x^*\right)$ is a boundary point of U.

(c) Let $\left(x^*, y^*\right) \in \mathbb{R}^{L\times I}_+ \times \prod_{j=1}^{J} Y_j$ be any Pareto efficient allocation. Show that it is a solution for the following optimization problem for some nonnegative weights $\boldsymbol{\lambda} = \left(\lambda_1, ..., \lambda_I\right) \left(\neq \mathbf{0} \in \mathbb{R}^L\right)$.

$$\max_{(x,y)\in A} \sum_{i=1}^{I} \lambda_i u_i\left(x_i\right)$$

2. Existence of Competitive Equilibrium

Consider a pure exchange economy $\mathcal{E}^{pure} = \left(\left\{\mathbb{R}^L_+, \succeq_i, e_i\right\}_{i=1,...,I}\right)$ (with free disposal) where $\succeq_i$ is continuous, monotone and strictly convex, and $e_i \gg 0$ for each i. Suppose that the market is regulated in this economy and there is a limit on the amount of each good which a consumer can consume. More specifically, let $K_\ell > 0$ be the limit per consumer for good ℓ for $\ell = 1, ..., L$. Assume that $K = (K_1, ..., K_L) \geq e_i$ for all i.

Given this restriction, consumer i's problem becomes

$$\max_{x_i \in \mathbb{R}^L_+} u_i(x_i) \text{ s.t. } p \cdot x_i \leq p \cdot e_i \text{ and } x_{i,\ell} \leq K_\ell \text{ for all } \ell,$$

where u_i is consumer i's utility function that represents $\succeq_i$. $(x^*, p^*) \in \mathbb{R}^{L \times I}_+ \times \mathbb{R}^L_+$ is a **competitive equilibrium** if (1) x^*_i is a solution for the above problem given p^* for each i and (2) $\sum_{i=1}^I x^*_i \leq \sum_{i=1}^I e_i$, where $\sum_{i=1}^I x^*_{i,\ell} = \sum_{i=1}^I e_{i,\ell}$ if $p^*_\ell > 0$ for any ℓ.

(a) Explain what is Walras' law and why it is satisfied in this economy even with this additional restriction on consumption.

(b) Show that there exists a competitive equilibrium in the above sense in this economy given any such $K \gg 0$.

(c) Show that a competitive equilibrium given K is in fact a genuine competitive equilibrium for the pure exchange economy $\mathcal{E}^{pure}$ without the consumption restriction if $K = (K_1, ..., K_L)$ is large enough.

3. Battle Royale: Cournot vs. Stackelberg

		Player 2	
		s	c
Player 1	S	5 , 2	3 , 1
	C	6 , 3	4 , 4

(a) Suppose the players move simultaneously. What are the Nash equilibria?

(b) Suppose Player 1 first chooses $a_1 \in \{S, C\}$, Player 2 sees 1's action and then chooses $a_2 \in \{s, c\}$. What are the SPNE?

Suppose Player 1 moves first, but that Player 2 observes 1's action with noise. In particular, Player 2 sees signal $\phi \in \{C, S\}$ such that

$$\Pr(\phi = S | a_1 = S) = 1 - \epsilon \qquad \text{and} \qquad \Pr(\phi = C | a_1 = C) = 1 - \epsilon.$$

(c) Draw the extensive form of the game.

(d) Suppose $\epsilon = 0$. What are the pure strategy weak-PBE? Are any of these sequential equilibria?

(e) Suppose $\epsilon \in (0, 1/4)$. What are the pure strategy weak-PBEs? Explain the difference between this and the last answer.

(f) Now consider the following mixed strategy PBE. Suppose Player 1 plays $a_1 = S$ with probability $\lambda \in (0, 1)$. Suppose Player 2 plays s with probability $\eta \in (0, 1)$ after signal $\phi = C$, and plays s with probability 1 after signal $\phi = S$. Let $\mu(\phi)$ be Player 2's belief that $a_1 = S$ after signal $\phi \in \{C, S\}$. Find the mixed strategy PBE.

4. Electronic Mail Game

Two armies are trying to coordinate an attack. The attack succeeds if both armies attack and the enemy is weak, but it fails if only one of them attacks or the enemy is strong. The payoff matrix is given by

$$\begin{array}{ccc|c|c|} & & \multicolumn{2}{c}{2} \\ & & \multicolumn{1}{c}{A} & \multicolumn{1}{c}{N} \\ \cline{3-4} 1 & A & x\ ,\ x & -1\ ,\ 0 \\ \cline{3-4} & N & 0\ ,\ -1 & 0\ ,\ 0 \\ \cline{3-4} \end{array}$$

where $x = 1$ if the enemy is weak and $x = -1$ if the enemy is strong.

Assume first that it is common knowledge that the enemy is weak, $x = 1$.

(a) What actions are rationalizable for either army?

Assume next that there is a (small) chance that the enemy is strong, $\Pr(x = -1) = \varepsilon \in (0, 1)$. Army 1 observes x. If $x = 1$, an e-mail is sent to army 2; however the e-mail only arrives with probability $1 - \varepsilon$ (if $x = 0$ no e-mail is sent). If army 2 receives the e-mail, a confirmation e-mail is sent to army 1; again the confirmation e-mail only reaches army 1 with probability $1 - \varepsilon$. Then the armies independently choose whether to attack.

(b) Argue that N is a strictly dominant strategy for army 2 if it does not receive the e-mail.

(c) Argue that N is the unique rationalizable strategy for army 1 if it knows that the enemy is weak, $x = 1$, but does not receive the confirmation e-mail.

(d) Now assume that whenever an army receives a confirmation e-mail, a confirmation e-mail is sent to the other army and arrives with probability $1 - \varepsilon$. After the end of the e-mail exchange (it ends in finite time with probability one) both armies independently choose whether to attack. Argue by induction that N is the unique rationalizable strategy for an army that has received the n^{th} e-mail but not the $n + 2^{th}$ e-mail (n is even for army 1 and odd for army 2).

5. Signaling

(a) Write down a simple Spence signaling model with T types.

(b) Consider $T = 2$. Characterize the complete set of separating PBE and pooling PBE.

(c) What is the Strong Intuitive Criterion? Which PBE satisfy this criterion? Explain carefully.

(d) For large T does the pooling equilibrium satisfy the Strong Intuitive Criterion?

Now consider a continuous version or the model where types have support $\Theta = [\alpha, \beta]$.

(e) Are there multiple pooling PBE? Explain. If there is a unique PBE, discuss whether it satisfies the Strong Intuitive Criterion. If there are multiple PBE, pick any one and discuss whether it satisfies the Strong Intuitive Criterion.

(f) The signaling cost function is $C(\theta, q) = \frac{q}{A(\theta)}$. A type $\theta \in [0, 1]$ worker has a marginal product $m(\theta) = \theta$. Characterize a separating signaling equilibrium as a solution to a differential equation.

(g) Suppose that an innovation halves the cost of education so that the new cost is $C(\theta, q) = \frac{q}{\hat{A}(\theta)} = \frac{q}{2A(\theta)}$. Draw a conclusion about the difference in equilibrium payoffs .

6. Optimal Auctions with Finite Types

Buyer i, $i = 1, 2$, has value $\theta_i = \Theta \in \{v_1, v_2\}$. Values are independently distributed . Let f_t be the probability that buyer i has value v_t. Consider the two type case where $(v_1, v_2) = (120, 200)$ and $(f_1, f_2) = \left(\frac{3}{5}, \frac{2}{5}\right)$.

(a) In the sealed second-price auction, what reserve price should the seller set in order to maximize expected revenue?

(b) Solve for the expected revenue maximizing selling scheme.

(c) How might this be implemented as an auction?

(d) Compare the buyer payoffs with those in the sealed second price auction. Is there revenue equivalence? If so why? If not why not?

(e) Describe a continuous c.d.f. $F(\theta)$ with p.d.f. $f(\theta)$ defined on $[120, 200]$ that is very similar to the cumulative probability for the two point distribution above. Without going into any technical details, discuss what you think the optimal auction will look like in this case.