

Thursday June 28

**UCLA
Department of Economics
Ph. D. Preliminary Exam
Microeconomic Theory
(Spring 2007)**

Instructions:

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE blue book to answer each question.

Section A.

1. Equilibrium over time

There are two consumers (or groups of identical consumers). A type h consumer has a utility function

$$U^h(x^h) = \alpha^h \ln x_1^h + (1 - \alpha^h) \ln x_2^h, \quad h \in \{A, B\}, \text{ where } \alpha^A = 3/4 \text{ and } \alpha^B = 1/4.$$

where $x^h = (x_1^h, x_2^h)$ is consumer h 's consumption of the single commodity in periods 1 and 2.

There is no storage. The aggregate endowment is $\omega = (\omega_1, \omega_2)$. Consumer A's endowment is a fraction λ of the aggregate endowment.

(a) Solve for the equilibrium spot-futures price ratio and show that it can be expressed as follows.

$$\frac{p_1}{p_2} = \frac{\alpha^\lambda}{1 - \alpha^\lambda} \frac{\omega_2}{\omega_1}, \text{ where } \alpha^\lambda = \lambda \alpha^A + (1 - \lambda) \alpha^B$$

(b) If $\lambda = 1/2$ and $\omega_1 = \omega_2$ what is the equilibrium interest rate? What are the range of possible interest rates as λ varies.

Henceforth assume that the aggregate endowment is the same in each period and that storage is costless.

(c) For what values of λ will the equilibrium involve storage? As λ varies between 0 and 1, what is the range of possible interest rates in this economy.

(d) If there is storage, solve for consumer A's equilibrium period 1 consumption as a function of λ .

2. Accumulating capacity with variable demand

A monopoly with installed capacity y_t in period t sells outputs $q_t = (q_{t1}, q_{t2})$ in two sub-periods. The marginal operating cost is c .

(a) Prove that if revenue in each sub-period $R_t(q_t)$ is concave, then the gross profit in the period

$$\Pi(y_t) = \max_{q_1, q_2} \{R_1(q_1) + R_2(q_2) - c(q_1 + q_2) \mid q_1 \leq y_t, q_2 \leq y_t\}$$

is also concave.

(b) Installed capacity depreciates at the rate θ . The monopolist can add $x(t)$ units of new capacity at a cost of $C(x_t)$ where $C(\cdot)$ is a strictly convex function. Thus $y_{t+1} = y_t(1 - \theta) + x_t$. The interest rate is r .

Analyze as completely as you can the optimal accumulation of installed capacity, starting from a capacity of zero.

(c) Suppose that demand in sub-period 1 is much higher than in sub-period 2. Explain why the price of the high demand commodity $p_1(q_{1t})$ will decline over time approaching some limiting price. What can you say about the time path of the price of the low demand commodity.

Section B

3. Auctioning Two Items

Two identical items will be sold to 3 bidders in a sealed bid auction in which the *two highest bidders* each win one item and *each pay the their own bid*. Bidders have private values, uniformly and independently distributed on $[0, 1]$. Find the unique symmetric equilibrium in non-negative, smooth strictly increasing bid functions.

4. Selling Three Cars

Three Sellers each have one car to sell to a single Buyer. All the cars come from one of two plants: at plant G, the fraction p of the cars produced are of High quality and the rest are of Low quality; at plant B *none* of the cars produced are of High quality, *all* are of Low quality. Each Seller observes the quality of his own car, but not which plant they come from. The probability that the cars come from a given plant is 0.5.

The Sellers value High quality cars at 10 each and Low quality cars at 5 each.

The Sellers and the Buyer have agreed to use the following mechanism: The Sellers each report the quality of their car. If two or more of them report High, Sellers who report High will receive 12 and Sellers (if any) who report Low will receive 11; if two or more of them report Low, Sellers who report Low will receive 10 and Sellers (if any) who report High will receive 9. (The Buyer has agreed to accept all cars at these prices.)

- (a) Consider the event that the other buyers observe H . Explain why the conditional probabilities of this event are $P(HH | H) = p^2$ and $P(HH | L) = [p(\frac{1-p}{2-p})]^2$, where a seller is conditioning on his own observation. Solve also for the conditional probabilities of the other events.
- (b) For $p = 0.9$, show that it is a Bayesian Nash equilibrium for all Sellers to report truthfully.
- (c) For $p = 0.5$, show that it is not a Bayesian Nash equilibrium for all sellers to report truthfully.
- (d) For $p = 0.5$, find a symmetric Bayesian Nash equilibrium in behavior strategies.

Section C

5. Investment under positively assortative matching

The attributes of buyers and sellers are represented by numbers $b_i, s_j \geq 0$, $i, j = 1, \dots, n$; and the value of a match is $V(b, s) = bs$. *Prior to matching*, each buyer and seller can invest in attributes at a cost of $c_i(b_i)$ and $c_j(s_j)$, where $c_i(0) = c_j(0) = 0$, all i and j . After attributes are chosen an optimal matching is made and *ex post* rewards are determined by selecting a dual solution to the LP problem in which buyer i receives q_i and seller j receives r_j . Denoting $\mathbf{b} = (b_1, \dots, b_n)$ and $\mathbf{s} = (s_1, \dots, s_n)$, the payoffs are $\pi_i(\mathbf{b}, \mathbf{s}) = q_i(\mathbf{b}, \mathbf{s}) - c_i(b_i)$ and $\pi_j(\mathbf{b}, \mathbf{s}) = r_j(\mathbf{b}, \mathbf{s}) - c_j(s_j)$. Rearrange the attributes, if necessary, so that $b_1 \leq b_2 \leq \dots \leq b_n$ and $s_1 \leq s_2 \leq \dots \leq s_n$.

- (a) Demonstrate that for all i , matching b_i with s_j when $j = i$ is optimal and that a dual solution satisfies $q_1 \leq q_2 \leq \dots \leq q_n$, $r_1 \leq r_2 \leq \dots \leq r_n$.
- (b) Ordinarily there are many choices for dual solutions. However, if (1) for every $i > 1$, either $b_i = b_{i-1}$ or $s_i = s_{i-1}$ and (2) either $s_1 = 0$ or $b_1 = 0$, show that there is only one dual solution. [Start from the bottom and work up.]

The possible attributes for buyers are $\{0, 2, 5\}$ and for sellers $\{0, 4, 8\}$. For each buyer $i = 1, 2, 3, 4$, $c_i(0) = c_i(2) = 0$ and $c_i(5) = 13$. For sellers $j = 1, 2$, $c_j(s)$ is very large for $s \neq 0$ (they are dummies), while $c_j(0) = c_j(4) = 0$ and $c_j(8) = 9$, for $j = 3, 4$. When all buyers choose $b_i = 2$ and sellers $j = 3, 4$ choose $s_j = 4$, the resulting choices satisfy the conditions in (b); hence, can be claimed to be perfectly competitive.

- (c) Show that at these choices, there are no individually profitable deviations, i.e., they constitute a Nash equilibrium.
- (d) Show that the equilibrium in (c) is not efficient.
- (e) Does there exist a division of the surplus for an efficient investment choice that is consistent with Nash equilibrium? Does there exist a division of the surplus for an efficient investment choice that precludes its compatibility with Nash equilibrium; hence, making the equilibrium in (c) unique?

6. Social opportunity cost

Utility for consumer i is $u_i(z_i, m_i) = a_i z_i - b_i z_i^2/2 + m_i$, where $a_i, b_i > 0$ and $z_i \geq 0$, $i = 1, \dots, n$. The money cost of supplying z is $cz^2/2$. At an efficient allocation, define the *marginal* social opportunity cost (SOC) of z_i as the cost to others of an infinitesimal change when i 's consumption of z_i increases infinitesimally, whereas the *individual* SOC of z_i is the (minimum) cost to others of i consuming z_i rather than 0, in which case the allocation is readjusted to be efficient for all individuals other than i . The commodity z may either be a private good or a (pure) public good with respect to consumption. [Clearly labeled diagrams can be a useful supplement to your answers.]

(a) If z is a private good, find the marginal SOC of z_i in terms of the parameters $\{(a_i, b_i)\}$ and c when $b_i = b$, all i . Write the expression for the individual SOC of z_i . If i pays the marginal SOC per unit for all units of z_i , how does that compare with individual SOC? [Suggestion: Look for efficient price.]

(b) If z is a public good, what is the marginal SOC in terms of the parameters when $b_i = b$. Does i pay the marginal SOC with Lindahl pricing of the public good? Write the expression for the individual SOC. [Suggestion: Look for efficient quantity.]

For the following, assume $a_i = a$, $b_i = b$ for all i , and the parameter c can vary with n .

(c) If z is a private good and $c_n = c/n$, demonstrate what happens to the relation between between marginal and individual SOC as $n \rightarrow \infty$.

(d) If z is public and $c_n = c$, demonstrate what happens to the marginal and individual SOC as $n \rightarrow \infty$?