UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2006)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Equilibrium under Uncertainty

There are two groups of individuals. The members of group A hold a period 1 endowment of 10 and shares in asset A that has a state dependent period 2 return vector $z_2^A = (10, 10, 6)$. The member of group B hold a period 1 endowment of 14 and shares in asset B that has a state dependent period 2 return vector $z_2^B = (6, 22, 10)$. The three states are equally likely. Goods are non-strorable. Each individual has the same logarithmic VNM utility function with discount factor δ, so expected utility of household h is

$$U^h(c^h) = \ln c_1^h + \frac{\delta}{3}(\ln c_{21} + \ln c_{22} + \ln c_{23}).$$

(a) If the spot price of the commodity is 1, solve for the Walrasian equilibrium state claims prices for the period 2 commodities as a function of the discount factor.

(b) If $\delta = 1$, solve also for the value of the two assets.

(c) If there are no state claims markets but individuals can trade shares in assets, is the allocation in (a) achievable?

(d) Suppose that the commodity can be costlessly stored. For what discount factors will there be no storage?

2. Input and Output Prices

(a) Show that the Cobb-Douglas production function $q = z_1^\alpha z_2^\beta$ has a cost function

$$C(q, r) = q^{\frac{1}{\alpha+\beta}} \left(\frac{r_1^\alpha r_2^\beta}{\left(\frac{\alpha}{\alpha+\beta}\right)^\alpha \left(\frac{\beta}{\alpha+\beta}\right)^\beta} \right)^{\frac{1}{\alpha+\beta}}$$

In a 2×2 economy with aggregate input vector $z = (1, 1)$ the production functions for the two commodities are: $q_A = z_{A1}^{1/2} z_{A2}^{1/4}$ and $q_B = z_{B1}^{1/2} z_{B2}^{1/2}$.

(b) Confirm that the first production function is strictly concave and the second is concave.

(c) If (q_A^0, q_B^0) and (q_A^1, q_B^1) lie on the boundary of the production possibility set prove that all convex combinations are in the interior.

(d) Show that if both commodities are produced, the input price ratio $r_1/r_2 \in [1, 2]$.

(e) Is there any output price vector (p_A, p_B) such that the economy will specialize in the production of commodity A?

3. Bargaining

Consider the following variation on ultimatum bargaining: there is a pie worth 10 dollars. Player 1 makes a proposal to divide the pie either $50-50$, or $60-40$ with the larger share going to himself. Player 2 can accept or reject. If he accepts the pie is divided as proposed; otherwise neither player gets anything. Let m_i be the monetary payoff to player i. Suppose that player i's utility is $m_i - cm_{-i}$, where $-1 \leq c \leq 1$. Suppose that it is observed that all $50-50$ offers are accepted and 60% of $60-40$ offers are accepted.

(a) Assuming that player rejecting and accepting offers strictly prefer to do so, how many types (distinct values of c) would you need to explain this fact.

(b) Suppose that one "type" has $c = 0$. Is this consistent with the facts? What fraction of the population must be this type?

(c) Based on the fact that offers of $60-40$ are rejected, what range must the other value of c lie in? What fraction of the population must be of this type?

(d) Which type makes 60-40 offers? What fraction of the offers must be 60-40 offers? What does this tell you about the value of by the second type?

(e) Suppose you were told that the same people who made $50-50$ offers also rejected $60-40$ offers (and in particular that only 40% of offers were for $50-50$). Is that consistent with this theory?

4. Risk Aversion

From experimental data of Peter Boessarts and William Zame, individuals in the laboratory are indifferent between getting nothing, and a gamble paying $15.00/p$, $-6.00/p$, 0.00 each with probability $1/3$, where p is an endogenously determined price.

For an individual with constant relative risk aversion, find the coefficient of relative risk aversion as a function of wealth, using the standard approximation. Suppose first that $p = 1.0$.

(a) If wealth is $350,000$, what is the coefficient of relative risk aversion?

(b) If the coefficient of relative risk aversion is 20, what is wealth?

(c) If preferences are logarithmic what is wealth?

(d) If wealth is 400 and preferences are logarithmic, what is p?

(e) Under what circumstances would (d) make sense?

5. Public versus Private Goods

Individuals face two kinds of choices. In one, the choices are PUB $= \{0, 1\}$ in which the color of the asphalt on streets will what it has been (0) or light green (1). In the second, the choices are PRI $= \{s_1, \ldots, s_n\}$, where $s_i \in \{0, 1\}$ and $\sum_i s_i = 1$. ($s_i = 1$ means that i gets a two-week (expenses paid) trip to Tokyo and everyone else stays home.) Normalize utility to be 0 whenever s or s_i is 0. Individual i's tastes is determined by the number w_i according to:

- $v_i(s) = w_i$, if $s = 1$ and $v_i(s) = 0$ if $s = 0$, when $s \in$ PUB
- $v_i(s) = w_i$, if $s = s_i = 1$, and $v_i(s) = 0$ otherwise, when $s \in$ PRI.

Hence, the characteristics of the economy are given by a vector $w = (w_1, \ldots, w_n)$ whether the choices are in PUB or PRI. For choices in PUB allow the possibility that $w_i < 0$, whereas for PRI assume $w_i \geq 0$. There are no cost differentials between choices in PUB or among those in PRI, so treat costs as zero.

(a) What are the conditions for an optimal choice for w in PUB? In PRI?

A *mechanism* can be regarded as a mapping from economies w to $(s(w), p_1(w), \ldots, p_n(w))$, where $s(w) \in$ PUB or PRI and $p_i(w)$ is the money payment made by i. The utility of the outcome to i is $v_i(s(w)) - p_i(w)$ assuming that everyone reports honestly, i.e., $v_i(s = 1) = w_i$.

(b) *Define* the conditions for a mechanism to encourage individuals to truthfully reveal their preferences. Are the conditions for PUB different from PRI? (Note: Here and below, ignore incentive compatibility conditions with respect to the supply of PUB or PRI.)

(c) Suppose a mechanism satisfies the optimality condition of (a). Give sufficient conditions for the mechanism to satisfy the incentive compatibility condition of (b). Outline an argument to justify your claim. Do the conditions vary between PUB and PRI?

Two conditions on a mechanism are: *weak budget balancing*, i.e., for all w, $\sum_i p_i(w) \geq 0$ (money payments not less than cost); and, *voluntarism*, i.e., for all w, $v_i(s(w)) - p_i(w) \geq 0$ (utility not less than *status quo*).

(d) Do the conditions for an optimal incentive compatible mechanism in (c) imply differences with respect to weak budget balancing and voluntarism for PUB compared to PRI? Explain.

6. Inter-regional Trade

There are R identical regions. In each region, there is a single household with utility function that distinguishes among $j = 1, \ldots, J$ different *types* of commodities and $k = 1, \ldots, K$ *varieties* within any type. The utility function is

$$u(x_{11}, \ldots, x_{jk}, \ldots, x_{JK}) = \sum_{j=1}^{J} \left(\sum_{k=1}^{K} (x_{jk})^{\beta} \right)^{1/2}$$

The parameter $1/2$ controls substitution across commodity types, whereas $\beta \in (1/2, 1)$ controls substitution among varieties within a type. Output of any variety of any good in any region is produced from labor according to the technology $Y = \{(y, \ell)\}$, where $\ell \geq 0$ is labor and $y = \ell - 1$, if $\ell \geq 1$ and $y = 0$, otherwise.

Each household (region) has $L = 2J$ units of labor. Assume K is large (but finite).

(a) The technology Y is non-convex. Describe in a diagram the smallest convex technology $\widehat{Y}$ such that $Y \subset \widehat{Y}$. Is there any scale at which the input/output combinations from Y coincide with those from $\widehat{Y}$?

(b) If the technology were $\widehat{Y}$, find Walrasian equilibrium in any region and show that there are no gains to inter-regional trade either in commodities or labor.

(c) With the technology Y, show that there would be inter-regional gains from trade in commodities, assuming that labor is immobile. In particular, letting g_2 be the maximum *per capita* gains from trade with two regions and g_1 the gains within a single region, show that $g_2 > g_1$.

(d) With technology Y, suppose $R = NJ$. What happens to $g_{NJ} - g_{(N-1)J}$ as N increases? Explain. As $N \to \infty$ does inter-regional trade permit an allocation that converges to the allocation in (b)? Does your answer depend on whether there is trade in labor as well as commodities?