UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2003)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Life Cycle Saving

A consumer lives for three periods. The present value of her financial assets and future earnings is W. Her utility function is $U(c) = u(c_1) + \delta u(c_2) + \delta^2 u(c_3)$, where $u(c) = c^{1/2}$. She can borrow and lend at the interest rate r.

(a) Write down the consumer's optimization problem and obtain the first order conditions.

(b) Define $\gamma = \delta^2(1+r)$ and show that the first order conditions can be written as follows.

$$\frac{1}{c_1} = \frac{\gamma}{\frac{c_2}{1+r}} = \frac{\gamma^2}{\frac{c_3}{(1+r)^2}}.$$

(c) Hence or otherwise show that her optimal first period consumption is $c_1^* = \frac{1-\gamma}{1-\gamma^3} W$.

(d) Explain why the solution is easily modified if there are T rather than 3 periods.

(e) With T large, what is the marginal propensity to consume out of an unanticipated in crease in the wage (i) in period 1 (ii) in period T, where T is large.

(f) What is the solution in the infinite horizon case (i) if $\gamma = 0.9$ (ii) if $\gamma = 1.1$?

2. Time and Uncertainty

There are N individuals all with the same two period preferences

$$U(c_1, c_{21}, c_{22}) = \ln c_1 + \sum_{s=1}^{S} \pi_s \ln c_{2s}$$

There are S states in period 2. Each individual holds *a* share of two assets. Asset *a* yields 100 in each period regardless of the state. Asset *b* yields 100 in period 1 and 100*s* in period 2 if the state is *s*..

Initially consider the case $S = 2$.

(a) Solve for the equilibrium state claims prices in terms of the probabilities (π_1, π_2).

(b) Hence solve for the equilibrium asset prices.

(c) If there are no state claims markets but individuals can trade shares in the two assets, how will this affect asset prices and consumption.

(d) Suppose next that there are three states and that the probability vector is (π_1, π_2, π_3). If the individuals can only trade in the two assets will they be better off, worse off or equally well off compared with the outcome when all state claims can be traded. If it is possible you should solve for the equilibrium asset prices.

3. **Long-run versus Short-Run**

Consider the following normal form in which the row player is long-run.

	a	b	c
A	1,0	0,6	2,5
B	0,6	1,0	9,5

If the long-run player's action is perfectly observed what is the best payoff to the long-run player in the repeated game, and for what discount factors?

4. **Signaling, Sequentiality and Dominance**

Consider the Cho-Kreps beer-quiche game. Nature moves first and chooses 50-50 between the types tough and wimp for player 1. Player 1 knows his type and moves second, and may choose beer or quiche. Player 2 moves last and knows whether player 1 chose beer or quiche, but not player 1's type. Player 2 chooses between exit and duel. If player 2 chooses exit, he gets 0. If he chooses duel, he gets +1 if he faces a wimp and –2 if he faces a tough type. Player 1 gets a sum of payoffs depending on his own choice and player 2's choice. A tough type gets 1 for beer, 0 for quiche; a wimp gets 0 for beer, 1 for quiche. Both types get in addition 0 if player 2 exits and –2 if player 2 chooses duel. Show that the only sequential equilibrium in which no player plays a weakly dominated strategy is for both types of player 1 to choose beer.

5. The Failure of Replica Invariance

Consider an economy with N individuals having identical concave tastes given by the quasi-linear utility function $v(z) + m$ and endowments that are identically distributed random variables W_i, $i = 1, \ldots, N$, e.g., taking the values 0 or 1 with equal probabilities. In this setting, expected utility Ev is the relevant notion.

(a) Assuming each vector of endowments $\omega_1(s), \omega_2(s), \ldots, \omega_N(s)$ is controlled by the state s, define price-taking equilibrium for this economy. (You might begin by assuming $N = 1$. How many prices are there?)

(b) What is the optimal allocation of the non-money commodity as a function of N and how does it compare with price-taking equilibrium?

For the following assume that all individuals have the same mean-variance preferences, i.e., $Ev(X) = E(X) - Var(X)$, where X is the individual's consumption as a random variable, E is the expectation and Var is the variance.

(c) Ordinarily, if there are N individuals with identical concave utility functions, there would be (I) no gains from trade. Also, if tastes were identical but endowments were not and the economy were replicated, (II) the maximum per capita gains would not change. Letting g_N be the maximum (expected) total gains from trade, show that (I') $g_2 > g_1$ and (II') $g_{2N} > 2g_N$.

(d) How can you explain that (I) and (II) are false and are replaced by (I') and (II')?

(e) Let Ev_i^N stand for expected utility received in price-taking equilibrium when there are N individuals. For any $N > 1$, show that the MP Inequality holds, i.e.,

$$\mathrm{MP}_i^N = g_N - g_{N\setminus\{i\}} > Ev_i^N.$$

(f) Why and in what sense does $\mathrm{MP}_i^N \to Ev_i^N$ as $N \to \infty$?

6. Smoking Regulations in Restaurants

Los Angeles City has made it illegal to smoke in restaurants. One commentator remarked: "This makes about as much sense as considering a ban on the kind of food (e.g., Greek, Italian, Indian, etc.) that can be served in restaurants. Granted that it is not practical for smokers to bribe non-smokers every time they light up, but such a remedy is unnecessary. If there is a demand for smoke-free restaurants, restaurant entrepreneurs will find it profitable to supply them."

Besides money there is one commodity, a standard restaurant meal of which each household can consume $z_h \in \{0, 1\}$. There are two kinds of households, smokers (s)

and non-smokers (ns). The commodity is further distinguished by α, the fraction of smokers in the restaurant where the meal is eaten. The utility function of a smoker is

$$v_s(z_s^\alpha) = \begin{cases} 10 & \text{if } z_s^\alpha = 1 \text{ and } \alpha > 0, \\ 0 & \text{if } z_s^\alpha = 1 \text{ and } \alpha = 0 \text{ or } z_s = 0 \end{cases}$$

Smokers do not enjoy eating out unless they can smoke; but as long as she can smoke, a smoker is indifferent to the number of other smokers. Non-smokers' utility declines with the fraction of smokers in the restaurant according to

$$v_{ns}(z_{ns}^\alpha) = \begin{cases} 10/(1+\alpha) & \text{if } z_{ns}^\alpha = 1, \\ 0 & \text{if } z_{ns} = 0 \end{cases}$$

All restaurants have the same technology given by the cost function

$$c_f(n_f, \alpha) = \begin{cases} (9 + (n_f)^2)(1 + .5\alpha) & \text{if } n_f > 0, \\ 0 & \text{if } n_f = 0 \end{cases}$$

where n_f is the number of standard meals. (There is a higher cost of producing a standard restaurant meal the greater the fraction of smokers, say because added precautions must be taken to prevent smoke odors from getting into the food.) For simplicity, assume that each firm supplies meals of only one type α. Further, assume that each restaurant has the right to exclude smokers.

(a) For any α, illustrate average costs? For any α, what is the "efficient scale" of a restaurant.

(b) Suppose the population of households consists of two smokers and one non-smoker and there is free entry in restaurants. What do you think the equilibrium allocation will be? Is it optimal? (Suggestion: Given free entry, will price exceed average cost?)

(c) Suppose there are 3 smokers and 3 non-smokers. What are the optimal and equilibrium allocations? Compare with part (b) and explain.

(d) If there are many smokers and many non-smokers, then no matter what the proportions of each, explain why — on the basis of the above model — it would be difficult to justify the need for an ordinance such as enacted in Los Angeles to forbid smoking in restaurants.

(e) What kinds of changes to the model would you suggest to justify that, on efficiency grounds, smoking should be banned in restaurants?