UCLA

Department of Economics

Ph.D. Preliminary Exam

Microeconomic Theory
(Spring 2002)

Instructions:

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any one question. Do NOT answer all the questions.
- Use a SEPARATE answer book for each question.

1. Short Takes

(a) Cournot duopolists face the demand price function $p = a - b(q_1 + q_2)$. Each firm has the same technology and both are price takers in input markets. Analyze as completely as you can the effect of a reduction in an input price on the Cournot equilibrium output price.

(b) "A strength of Walrasian equilibrium theory is that it can be so easily extended to explain choice over time and choice under uncertainty." Comment.

(c) "If all individuals have the same homothetic preferences, market demand is very easily characterized." Prove this or explain why it is false.

2. The Pareto Paradox

game 1	A	B
A	5,5	2,7
B	7,2	3,3

game 2	A	B
A	4,4	1,3
B	3,1	0,0

Which game would you prefer to play and why? What is paradoxical about your answer? Now focus on game 1, and suppose that the game is played twice (no discounting) and that a fraction p of the players are "irrational" in the sense that they play A in period 1 and in period two play the same strategy their opponent played in period 1. How large would p have to be for there to be an equilibrium in which all players rational or "irrational" cooperate in period 1?

3. Convergence with Fixed Costs

Each of I identical consumers has quasi-linear utility $q_i - q_i^2/2 + m_i$ and each of J producers has cost $q_j^2/2 + F$ if $q_j > 0$ and 0 otherwise. (All quantities are non-negative.) Note: Even though identical, technologies are proprietary.

(a) Letting $Q = \sum q_i$, show that the inverse demand is $P_I(Q) = 1 - Q/I$.

(b) With $F > 0$, producer's costs are U-shaped. Consequently, a price-taking equilibrium need not exist. Nevertheless, show that when $I = J = 1$, if $F = 1/8$ a price-taking equilibrium does exist and that producer's profits are zero.

(c) If a price-taking equilibrium exists for $I = J = r$, replicating that k times would evidently be a price-taking equilibrium for $I = J = kr$. Use this fact to explain that when producers have identical U-shaped costs curves, if a price-taking equilibrium exists, profits must be zero. Under what conditions is the marginal product of a producer also zero. In this example, is there an r for which those conditions are fufilled?

(d) When $F = 1/8$ and $I = J = r$, find the symmetric Cournot equilibrium. (Suggestion: Write j's profit as $\pi_j(q_j, Q_{-j})$, where $Q_{-j} = Q - q_j$. By symmetry, $Q = Jq$ and therefore $Q_{-j} = (J-1)q = (I-1)q$.) The Cournot equilibrium q for each producer will depend on r.

(e) What happens to prices, quantities and profits in Cournot equilibrium as $r \to \infty$?

4. 2×2 economy

Outputs are produced using labor and capital according to the following production functions:

$$X_A = L_A^{\frac{1}{4}} K_A^{\frac{1}{4}} \text{ and } X_B = L_B^{\frac{1}{2}} K_B^{\frac{1}{2}}.$$

The economy has 200 units of Labor and 50 units of capital.

(a) Characterize as completely as you can the set of feasible outputs for this economy.

(b) Outputs can be freely traded at world market prices (P_A, P_B). For what prices would the economy specializes in production of (a) commodity A, (b) commodity B.?

(c) If the economy is closed and preferences are given by the utility function $U(X_A, X_B) = 2\ln x_A + 2\ln X_B$, solve for the Walrasian equilibrium.

5. The First and Second Theorems of Welfare Economics

In a double-auction model in which each buyer or seller trades at most one unit of a discrete homogeneous commodity, the population of buyers' and sellers' reservation values are each uniformly distributed on $[0,1]$. (The total population of both buyers and sellers therefore has mass 2.)

(a) Illustrate market demand and supply and find equilibrium price and total quantity bought and sold.

(b) A buyer with characteristics v envies a buyer v' if v would prefer the allocation of non-money and money commodities that v' receives to the allocation v is receiving. (A similar statement applies to a seller with characteristics c.) Show that money payments made in the equilibrium in (a) lead to no envy. Is this because each individual is infinitesimal? Is this result limited to the double-auction model?

(c) The equilibrium in (a) has some individuals who gain nothing from trade while others gain varying amounts. In the interest of egalitarianism with respect to gains from trade, what is the maximum per capita gains? Call it e.

(d) The egalitarian outcome could be obtained by modifying the result in (a) as follows: If an individual gains g in the equilibrium of (a), impose a tax of $g-e$ (when $g \geq e$) paid in the money commodity and use the proceeds to bring everyone else up to e. If the taxing authority knows only the distribution of individuals' characteristics but not who is who, show that egalitarian taxation produces envy. Be explicit as to who would envy whom.

(e) Envy leads to misrepresentation and failure to implement the above tax scheme. Is there is any taxation scheme that would simultaneously achieve an egalitarian distribution of the gains from trade, efficiency, and no envy? If there is, what is it? If there is not, why not?

6. Long-run Short-Run with Noise

Consider the following normal form in which the row player is long-run.

	a	b	c
A	0,0	2,3	0,4
B	0,4	3,3	0,0

If the long-run player's action is perfectly observed what is the best payoff to the long-run player in the repeated game, and for what discount factors?

Now suppose that the long-run player's action is observed with error. Specifically, if he chooses A, subsequent short-run players observe A with probability .9 and B with probability .1; if he chooses B, subsequent short-run players observe B with probability .9

and A with probability .1. Note that the normal form does not change, just the information available to subsequent short-run players after each match. Now what is the best payoff to the long-run player in the repeated game and for what discount factors?