UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2001)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Life-cycle consumption

A consumer can borrow and lend at the same interest rate r. The consumer's total wealth (human + financial) growth equation is:

$$A_{t+1} = (1+r)(A_t - c_t).$$

The consumer's lifetime utility function is:

$$U = \sum_{t=1}^{T} \delta^{t-1} u(c_t), \text{ where } u(c) = \ln c.$$

(a) If the consumer must leave assets A_{T+1}, write the optimization problem and obtain the necessary conditions for an optimum.

(b) Sketch a proof that the indirect utility function $V(A_{T+1})$ is concave. Depict the indirect utility function in a carefully drawn figure and use it to obtain an upper bound for the consumer's lifetime (T period) utility.

(c) Consider the infinite horizon version of this problem and show that $c_t = (1-\delta)A_t$ is feasible.

(d) Using your answers to (b) and (c) (or otherwise), show that this path is a solution to the infinite horizon problem.

(e) What can you say about the effect on first period consumption if A_1 increases for: (i) the infinite horizon problem; (ii) the finite horizon problem?

2. Joint costs and fixed costs

A lumber company brings logs to its mill at a cost of $60L$, where L is the number of logs. Each log yields one unit of three potentially marketable products. The unit variable costs of finishing and marketing the three products are

$$c_1 = 30, \quad c_2 = 20, \quad c_3 = 40.$$

Demands for the three products are:

$$p_1 = 130 - q_1, \quad p_2 = 120 - 2q_2, \quad p_3 = 200 - q_3.$$

(a) Solve for the profit maximizing outputs and prices of the three products.

Suppose that there are also fixed costs associated with marketing each product. The fixed costs are

$$(F_1, F_2, F_3) = (1000, 1500, 2000).$$

(b) What will the profit maximizing choices be?

(c) If F_3 increases to 3500, how will that influence the firm's profit maximizing production plan?

(d) How would your answer to (b) be affected if logging costs were $35L + L^2/4$, rather than $60L$?

3. Long-run short-run with noise

A short-run supplier has the option of supplying a single indivisible item to a long-run firm. The firm has the option of paying for the item or not. If the firm pays, there is a 25% chance that the check will be lost in the mail. (Note: if the check is lost, the supplier does not receive the payment, *and* the firm is not charged for the item.) The firm values the item at \$5.00, and the supplier values the item at \$1.00. The payment is \$4.00, and both parties are risk neutral.

Find the best perfect equilibrium for the firm (of the infinitely repeated game with public randomization) as a function of the discount factor, first, assuming that the supplier can observe whether or not the check is lost in the mail and, second, assuming that the supplier can only observe whether or not payment is received.

4. Incomplete information bargaining

Consider an ultimatum bargaining game in which there are three dollar bills to be divided. (Note: four possibile proposals: $0, 1, 2, 3$.) There are two types of proposer. One is the normal type; the other is the "punishment" type who will hold the second player up to public ridicule (utility of -1) for accepting any offer. The type of proposer is private information.

(a) Analyze the sequential equilibria of this game.

(b) Do the beliefs supporting these equilibria make intuitive sense to you?

5. Property rights to innovations

Each of two firms, called I and II, can engage in R&D at a cost of C. A successful outcome by either one or both firms is worth V (its value to a third party). An unsuccessful outcome has zero value. The probability of success is $1/2$ for each firm.

(a) What is the (expected) total gain if only one firm engages in R&D (g_1)? If both firms engage in R&D (g_2)? Defining $\text{MP}_i = g_2 - g_1$, $i = I, II$, compare $\text{MP}_I + \text{MP}_{II}$

with g_2. Explain why it differs from the usual comparison. To achieve efficiency, should one or both firms engage in R&D when $\text{MP}_i > 0$?

(b) Without patent protection, if one firm (the innovator) engages in R&D and is successful, it receives βV, where $\beta \in [1/2, 1)$, while the other firm (the imitator) can costlessly reverse engineer and receives $(1-\beta)V$. Assume $\text{MP}_i > 0$ and $C > (1-\beta)V/4$. Show that a non-cooperative equilibrium cannot be efficient.

(c) Patent protection effectively means that $\beta = 1$. [Assume that if both firms engage in R&D and both are successful, each has a $50-50$ chance of getting the patent.] Under the assumptions in part (b), show that non-cooperative equilibrium with patent protection leads to efficiency. Patent protection is like "owner keeps all." Why does that work here?

(d) Suppose there is patent protection, but $\text{MP}_i < 0$ and $C < 3V/8$. Does non-cooperative equilibrium lead to efficiency? Explain.

6. Resource allocation for a public park

A fixed number of acres of land is to be made into a public park with two possible uses, as sports fields and botanical gardens. Let $x \in [0,1]$ be the fraction of the land devoted to sports. Costs are the same no matter what the value of x, and for purposes of simplification assume they are *zero*. There are n users of the park, each with a quasilinear utility function $v_i + m_i$, where $v_i = a_i x - b_i x^2$, and $a_i, b_i \geq 0$. (Not everyone is a sports enthusiast in the sense that v_i is not necessarily increasing in x.)

(a) Set up the problem defining an efficient choice of x for $\mathbf{v} = (v_1, \ldots, v_n)$ and, assuming an interior solution, use it to derive the first-order conditions.

(b) Describe a price-taking (Lindahl) equilibrium for this problem where individuals have to 'buy' x. Show that this price-taking equilibrium will satisfy the first-order conditions of (a).

(c) Assuming that x is efficient given individuals' announcements of their tastes, define a money payment scheme that encourages individuals to reveal their tastes (i.e., their a_i and b_i) truthfully, and which also has the property that if everyone's tastes are the same, money payments are zero. Show why your scheme works.

(d) Both the Lindahl and efficient incentive compatible mechanisms agree on $x(\mathbf{v})$ given the *announced* $\mathbf{v}$, but they differ in terms of the value of money payment $m_i(\mathbf{v})$. Write $m_i^L(\mathbf{v})$ as the money payment in the Lindahl mechanism and $m_i^L(\mathbf{v}) + \Delta_i(\mathbf{v})$ as the money payment in the incentive compatible mechanism. What is $\sum_i \Delta_i(\mathbf{v})$, i.e., the cost of providing individuals with the incentive to reveal truthfully?