

UCLA
Department of Economics
Ph. D. Preliminary Exam
Micro-Economic Theory
(SPRING 2000)

Instructions:

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Wealth Accumulation

Ali can borrow or lend at the interest rate r . She has an initial capital K_1 and an income stream (y_1, y_2, \dots, y_T) . Her preferences can be represented by the CES utility function

$$U(C) = u(C_1) + \beta u(C_2) + \dots + \beta^{T-1} u(C_T),$$

where $u'(C) = 1/C^{1/\sigma}$ and $\beta(1+r) > 1$.

Given a consumption vector (C_1, \dots, C_T) , Ali then accumulates capital according to

$$K_{t+1} = (1+r)(K_t + y_t - C_t).$$

(a) Show that total wealth $W_t (= K_t + y_t + \frac{y_{t+1}}{1+r} + \dots)$ accumulates according to

$$W_{t+1} = (1+r)(W_t - C_t)$$

(b) Show that Ali's optimal consumption vector must satisfy

$$\frac{C_{t+1}}{C_t} = (1+g),$$

where g depends only on r and the CES parameter σ .

(c) Draw a phase diagram with W_{t+1} on the horizontal and C_t on the vertical axes.

(d) Depict the optimal path $\{W_{t+1}, C_t\}_{t=1, \dots, T}$. In the special case where $K_1 = 0$, $y_t = \bar{y} > 0$ for $0 < T_1 \leq t \leq T_2 < T$ and $y_t = 0$ otherwise, describe capital accumulation over the life-cycle.

(e) How would the answer to part (d) change if Ali planned to leave a bequest B_{t+1} in period $T+1$?

2. 2×2 Economy

Output of commodity i , $i = 1, 2$ can be produced according to the constant returns-to-scale Cobb-Douglas production function $q_i = L^{\alpha_i} K^{1-\alpha_i}$, where $1 > \alpha_1 > \alpha_2 > 0$.

(a) Show that if the unit input prices are w and r , the cost function for the production of commodity i is

$$C_i(q_i, w, r) = q_i[r(w/r)^{\alpha_i}g(\alpha_i)], \text{ where } g(\alpha_i) = [\alpha_i^{\alpha_i}(1 - \alpha_i)^{1-\alpha_i}]^{-1}.$$

(b) Suppose the total endowment of the two inputs is $(\bar{L}, \bar{K}) = (100, 100)$. Around the production possibility frontier, as q_1 increases, what happens to the equilibrium wage-rental ratio? Explain carefully.

(c) What is the relation between the Walrasian equilibrium output prices and input prices?

(d) What is the range of possible equilibrium wage-rental ratios in this economy?

(e) Over what range of output prices is the economy specialized in the production of commodity 1?

3. Signaling Quality

I am thinking about having some landscaping work done at my house. I know that some gardeners are High quality and some are Low quality, but I cannot observe which. However, I can observe whether or not the gardener gives me a free sample (trimming a tree at my office) or not. After I observe whether or not the gardener gives me a free sample, the gardener sets a price for his landscaping work. (Each gardener sets the best price he can.) Having observed all this, I can accept or refuse. All the following is common knowledge:

- 1/2 of gardeners are High quality; 1/2 of gardeners are Low quality
- the free sample (trimming a tree at my office) costs the High quality gardener c_H and the Low quality gardener c_L
- the free sample (trimming a tree at my office) is worth V to me
- the cost of doing the landscaping work at my house is C (same for High and Low quality gardeners)
- the value to me of the landscaping work at my house is v_H if the gardener is High quality, v_L if the gardener is low quality; $v_H \geq v_L \geq C$ (the value to me of the work with the free sample is $v_H + V$ if the gardener is High quality, $v_L + V$ if the gardener is Low quality)

Analyze this situation as a game of incomplete information. In particular:

- (a) Make a careful sketch of the game tree, showing choices and payoffs.
- (b) Find inequalities among these parameters v_H, v_L, V, c_H, c_L, C which are necessary and sufficient for there to exist a completely separating sequential equilibrium (in pure strategies) in which both gardeners work. Find the equilibrium, including prices and beliefs.
- (c) Find inequalities among these parameters v_H, v_L, V, c_H, c_L, C which are necessary and sufficient for there to exist a completely pooling equilibrium (in pure strategies) in which both gardeners offer a free sample and both gardeners work. Find the equilibrium, including prices and beliefs.

4. Repeated Games

ROW and COL play the following game N times using the discount factor δ , $0 < \delta < 1$. For each $N = 1, 2, 3, 4$ find, as a function of δ , the largest number of times that (D, L) can be played as part of a subgame perfect equilibrium in pure strategies. In each case, give a careful description of the equilibrium strategies.

	L	R
U	3 0	0 0
D	2 2	0 3

5. Household Services versus Pay TV

HOUSEHOLD SERVICES: Each buyer $b = 1, \dots, B$ can spend at most one hour a day at home consuming personal services such as physical therapy, home repair, etc. Each seller $s = 1, \dots, S$ has up to one hour's capacity to provide his service to buyers: v_{bs} is b 's valuation/hour of the services of s and c_{bs} is the cost/hour to s of delivering services to b .

(a) Describe price-taking equilibrium in this market and demonstrate its efficiency.

PAY TV: Each viewer b can spend at most one hour a day watching TV. There are S different stations and v_{bs} is the per hour benefit to b of watching station s . Station s can supply up to one hour of programming—simultaneously to every buyer—at a cost of σ_s /hour. To guarantee exclusion, if station s produces $0 \leq y_s \leq 1$ units of programming, the station can choose to supply anything up to y_s units to any viewer.

(b) Formulate the price-taking equilibrium version of the pay TV market. Efficiency?

(c) One possible meaning of the term “perfectly competitive” is that there are no bargaining problems—in particular, there is no incentive to misrepresent one's characteristics to secure a more favorable outcome. Does the presence of alternative stations permit the market for pay TV to be as competitive as that for personal services? (Suggestion: Compare the amount a buyer pays in (a) and (b) with the “social opportunity cost” of the purchase.)

6. Consequences of Non-convexity

The utility function of each household $i = 1, \dots, N$ for commodities $k = 1, \dots, K$ is

$$U(x_{i1}, x_{i2}, \dots, x_{iK}, m_i) = \sum_{k=1}^{k=K} v(x_{ik}) + m_i$$

Each individual is endowed with *one* unit of labor (which is not an argument of the utility function) as well as an unlimited supply of the money commodity. There is an unlimited number of potential firms, each with a production function for producing any commodity k given by $x_{jk} = f(\ell_{jk})$, where ℓ_{jk} is the amount of labor used by firm j in producing commodity k . Therefore an economy is effectively described by (N, K, v, f) .

Assume that $v(x_{ik}) = 2(x_{ik})^{1/2}$. The function f can have one of the following forms:

(A) $x_k = (\ell_k)^2$

(B) $x_k = \begin{cases} \max\{2\ell_k - 10, 0\} & \text{if } 0 \leq \ell_k \leq 10, \\ \ell_k & \text{if } \ell_k \geq 10 \end{cases}$

(a) Provide a demonstration that if f is of the form (A), there is no price-taking equilibrium. Does this conclusion change if the size of the economy, in terms of the number of households N , is large?

(b) Provide a demonstration that if f is of the form (B), there is a price-taking equilibrium when N is large. (What is the equilibrium ratio of the price of labor to the wage rate? Although it is not essential, you may wish to set $K = 2$.)

(c) Suppose f is of the form (B) and $N = K$ and both are large. Show that there is no price-taking equilibrium.

(d) How do you explain the difference between (b) and (c)?