UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2015)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Representative Producer

Consider n producers with production possibility set $Y_j \subset \mathbb{R}^L$, $j = 1, ..., n$. Each Y_j is nonempty and closed. Let $\pi_j(p) = \sup_{y_j \in Y_j} p \cdot y_j$ be the maximum profit and $y_j(p)$ is the set of optimal productions (which may be empty) given price vector $p \in \mathbb{R}^L$. Also consider a hypothetical producer who has an access to all these production possibility sets in the sense that he is equipped with a total production possibility set $Y = \sum_{j=1}^{n} Y_j$. Let $\pi(p)$ and $y(p)$ be the profit function and the optimal production correspondence with respect to Y respectively. Answer the following questions.

(a) Suppose that each Y_j is convex. Does this mean that Y is convex? If so, prove it. If not, find a counterexample. Also do the converse. Suppose that Y is convex. Does this mean that each Y_j is convex?

(b) Suppose that Y is bounded above, i.e. there exists $\overline{y} \in \mathbb{R}^L$ such that $\overline{y} \geq y$ for any $y \in Y$. Show that $y(p)$ is nonempty given any $p \in \mathbb{R}^L_{++}$.

(c) Show that $\pi(p) = \sum_{j=1}^{n} \pi_j(p)$. Also show that $y(p) = \sum_{j=1}^{n} y_j(p)$ when $y(p)$ is nonempty (convexity/boundedness is not assumed for this question).

2. Revealed Preference

Suppose that you collect a finite data of price-consumption vector pairs $D = \{(p^t, x^t) \in \mathbb{R}^L_{++} \times \mathbb{R}^L_{+}, t = 1, ..., T\}$ by asking the same consumer which consumption bundle he would chose if the prices were $p^t, t = 1, ..., T$. Assume that his choice would be based on his preference $\succeq$ and his answer is honest.

D satisfies WARP if x^t is revealed preferred to x^s (i.e. $p^t \cdot x^t \geq p^t \cdot x^s$ and $x^t \neq x^s$), then x^s is not revealed preferred to x^t for any s, t. D satisfies GARP if x^t is indirectly revealed preferred to x_s (i.e. x^t is revealed preferred to x^{t_1}, x^{t_1} is revealed preferred to x^{t_2}, ... and x^{t_K} is revealed preferred to x^s), then x^s is not strictly revealed preferred to x^t (i.e. $p^s \cdot x^t \geq p^s \cdot x^s$) for any s, t. Answer the following questions.

(a) Suppose that this consumer maximizes the following utility function $u(x) = \min\left\{\sum_{\ell=1}^{L} \alpha_\ell x_\ell, \sum_{\ell=1}^{L} \alpha'_\ell x_\ell\right\}$ for some parameters $\alpha, \alpha' \in \mathbb{R}^L_{++}$. Discuss whether D must satisfy WARP/GARP or not in this case. If it may not satisfy either WARP or GARP, find such an example of D.

(b) Suppose that this consumer chooses optimally with respect to the following lexicographic preference: $x \succeq x'$ if and only $x_\ell \geq x'_\ell$ for $\ell = 1, ..., L'$ and $x_{L'+1} > x'_{L'+1}$ for some $L' \in \{1, ..., L\}$. Discuss whether D must satisfy WARP/GARP or not in this case. If it may not satisfy either WARP or GARP, find such an example of D.

(c) Discuss briefly why no lexicographic preference would be needed to rationalize D even if this consumer's true preference is the lexicographic preference described in (b).

3. Bayesian Games

Rose and Carl live in NY City and use the subways to play an ongoing game of tag. Rose takes the 1st Avenue subway and gets off at either the Market Street station or the South Street station. Carl will go to either Market Street or South Street; he can go to either station – but not both. If Carl goes to the wrong station, he will *never* tag Rose; if he goes to the right station *and* the station *is not crowded* he will tag rose for sure but if he goes to the right station and the station *is crowded* he will only tag Rose 50% of the time.

Rose and Carl agree that not tagging is worth $+10$ to Rose and -10 to Carl, and that tagging is worth -10 to Rose and $+10$ to Carl. Riding the subway is unpleasant; the extra cost to each of them to go to Market Street rather than to South Street is -2, no matter what else happens.

It is common knowledge that each station will be busy 50% of the time and that the conditions at the two stations are independent. Carl cannot see the conditions before he makes his decision where to go. Rose can see the condition at the South Street station (which is the first stop) but not the Market Street station *before* she decides where to get off

(a) Model this situation as a Bayesian Game and identify all the elements.

(b) Find all the Bayesian Nash Equilibria. (You may find it easier to solve in behavioral strategies).

4 Repeated Games

For the stage game G below, consider the infinitely repeated game $G^{\infty}(\delta)$ in which players use the discount factor $\delta \in (0, 1)$.

	L	R
U	3,3	0,0
D	2,6	6,2

(a) What is the pure strategy minmax payoff for each player? Explain why each player's equilibrium payoff cannot be below his minmax payoff.

(b) For $3 < x < 5$, the Folk Theorem tells us that there is a discount factor δ for which the long term average payoff $(x, 8 - x)$ can be supported as a subgame perfect equilibrium (in pure strategies). Let $\delta(x)$ be the *smallest* discount factor with this property. Prove that

$$\lim_{x \to 5-} \delta(x) = 1$$

(It is <u>not</u> necessary to compute δ explicitly or to find a subgame perfect equilibrium that achieves $(x, 8 - x)$.)

5. Bidding for Contracts

There are two qualified airport construction companies. The cost of building a new airport for firm i is c_i billion dollars. The cost for firm i is known only to this firm. However it is common knowledge that this cost is uniformly distributed on $[1, 2]$, that is, $F_i(c) = \Pr(c_i \leq c) = c - 1$, $1 \leq c \leq 2$ for every i.

The government agency invites each firm to submit a price p_i. The firm submitting the low price wins the contract and is paid the submitted price upon completion of the project. In the case of a tie, the winner is selected at random.

(a) Obtain an expression for the marginal informational rent of firm i.

(b) Solve for the symmetric equilibrium bid function $P(c_i)$.

(c) Politicians insist that the outcome should be efficient. Is the proposed scheme efficient? Is there any other scheme that is also efficient and also less costly for the agency? Provide as complete an explanation as possible.

(d) Suppose that the government announces that the project will only be built if there is a bid below 1.5 billion dollars. Solve for the new equilibrium bid function.

6. Spence Signaling Model

A type θ worker has marginal product $m(\theta, z) = \theta$, where z is his education. The worker's cost of education z is $C(\theta, z) = \frac{3z}{\theta}$. The worker's outside opportunity yields a payoff $u_o(\theta) = 2$.

(a) Characterize all the separating PBE if $\theta \in \{1, 3\}$ and there are more low quality workers than high quality workers.

(b) Under these assumptions are there any other PBE?

(c) Which of the PBE characterized in (a) and (b) satisfies the Cho-Kreps Credibility test? Explain carefully.

Suppose, henceforth, that worker types are continuously distributed on $[1, 3]$.

(d) Is there a separating PBE in which the signaling set is $S = [2, 3]$ and all other types prefer their outside opportunity?

(e) Are there other separating PBE? Explain carefully.

(f) Which of these PBE satisfy the Cho-Kreps Intuitive Criterion? Explain.