UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2007)

Instructions:

- You have **4** hours for the exam

- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

- Use a SEPARATE bluebook to answer each question.

1. Risk aversion

In a single commodity economy, Alex and Bev have strictly increasing Von Neumann-Morgenstern utility functions $V_A(c)$ and $V_B(c)$. Consider the following two definitions of "more risk averse"

D1: Bev is more risk averse than Alex if $V_B(c) = f(V_A(c))$ and $f(\cdot)$ is strictly concave.

D2: Bev is more risk averse than Alex if $-\frac{V_B''}{V_B'} > -\frac{V_A''}{V_A'}$.

(a) Show that D1 implies D2

(b) Explain why for any such pair of increasing functions for which V_B is more risk averse than V_A there is an increasing mapping f such that $V_B = f(V_A)$. Hence or otherwise show that D2 implies D1.

Henceforth consider a 2 state world.

(c) For any riskless consumption $\bar{c}$ there is a set of acceptable risks, that is, prospects preferable to c. Show that B's set of acceptable risks is a subset of A's set of acceptable risks.

(d) For any prospect $(c_1, c_2; \pi_1, \pi_2)$, a risk averse individual is willing to pay a risk premium x in order to replace the risk by its expected value, that is $V(\bar{c} - x) = \pi_1 V(c_1) + \pi_2 V(c_2)$, where $\bar{c} = \pi_1 c_1 + \pi_2 c_2$. For small risks appeal to Taylor's Expansion to characterize x and hence show that x is higher for Bev.

2. Constant returns to scale 2×2 economy with different input endowments

Countries A and B have the same CRS technology and every consumer has the same homothetic preferences. The production functions are $x_A = z_1^{1/2} z_2^{1/2}$ and $x_B = (z_1^{1/2} + z_2^{1/2})^2$. Country A has an input endowment of $\bar{z}_A = (100, 400)$. Country B has input endowment $\bar{z}_B = (400, 100)$. Initially both economies are closed economies.

(a) As the output of x_A rises along the set of production efficient allocations in country A, show that the MRTS must rise. (You must derive your results. It is not enough to look at endpoints.)

(b) Depict the set of feasible input vectors in a neat figure, showing both extreme points.

(c) Show that if preferences shift in favor or commodity A, the relative price of commodity A will rise and the shift in the relative price of input 1 can be signed.

(d) Repeat the analysis for country B.

(e) Under free trade will there be any trade? Explain.

3. Bayesian Nash equilibrium

Lance Armstrong and Floyd Landis agree to an exhibition bicycle race. Whoever trains hardest will win a prize they each value at $V = 2$, but training is expensive: cost $= ct$ where $t \in [0,1]$ is the level of training chosen and $c \in [1,2]$ is individual specific marginal cost. Lance and Floyd each know their own marginal cost of training and behave as if the other's marginal cost is uniformly distributed on $[1,2]$.

(a) Show that, whatever strategy Lance follows, Floyd's best response is <u>weakly decreasing</u> (in his own cost).

(b) Find a symmetric Bayesian Nash equilibrium in smooth strictly decreasing strategies. (Suggestion: it may help to think of this as an auction in disguise.)

4. A bridge building mechanism

The government of Ketchikan, Alaska is contemplating building a footbridge to a small island that has only three residents. The bridge would cost 30 (thousands of dollars); the cost must be shared by the residents. (The government is not allowed to subsidize construction or to have any money left over.) Each of the residents knows his/her own valuation for (use of) the bridge, but not the valuation of the other residents; the government does not know the valuation of any resident. The government and the residents each think the private valuations are either High or Low with equal and independent probability; for Residents 1 and 2, the High valuation is 10 and the Low valuation is 0; for Resident 3 the High valuation is V and the Low valuation is 0. (V is a parameter of the problem.)

For what values of the parameter V can the government find efficient (i.e., the bridge is built exactly when the social value is at least the cost), individually rational (i.e., all residents are willing to participate), incentive compatible mechanism? (You can assume the mechanism treats residents 1 and 2 symmetrically.)

5. Two models of decreasing costs

The primary reason why competition cannot be relied upon in neo-classical theory is decreasing costs. Advocates of regulation recommend marginal cost pricing which, under decreasing costs, leads to negative profits; hence, the need for subsidies financed by taxation. Anti-regulation economists point to the flaws in marginal cost pricing and the fact that pro-regulation economists underestimate private enterprise.

Suppose consumer i's utility is $u_i(q_i, m_i) = a_i q_i - b_i q_i^2/2 + m_i$, $i = 1, \ldots, n$. In model I, the costs of production are

$$c_I(q_1, \ldots, q_n) = F + \sum_i q_i,$$

if $\sum_i q_i > 0$ and 0, otherwise. In model II, the costs are

$$c_{II}(q_1, \ldots, q_n) = \sum_i c(q_i),$$

where $c(q_i) = f + q_i$, if $q_i > 0$, and 0, otherwise. $F, f > 0$. For either model, assume that there are several producers with identical costs. To further simplify, suppose $(a_i, b_i) = (\beta_i + 1, \beta_i)$, $\beta_i > 0$ and there are two consumers $(n = 2)$.

(a) In model I, find conditions on the parameters β_1, β_2 and F such that marginal cost pricing would be inefficient?

(b) In model I, show that the ability to engage in price discrimination over quantities and over consumers would allow a producer to determine whether or not output should be supplied — when β_1 and β_2 are known to the producer.

(c) In model II, suppose β_i is known only to the consumer i. Explain why competition would lead to efficient outcomes involving discriminatory pricing if output is supplied and why it would also be efficient if output were not supplied.

(d) Model II has been used as an argument against regulation. Should it be, or is there a relevant difference between the two models of decreasing costs?

6. Malicious advertising

There are 6 firms and 3 consumers. Each firm can produce at most one unit of product; the MC of firms I and II is 2, while the MC for the other firms is 4. Each consumer wants at most one unit and their tastes are identical. Firms I and II can advertise (A) at a cost of 1, or not advertise (NA). If neither advertises, each consumer values each firm's product at 7. If both firms advertise, the countervailing ads cancel each other out and consumers value each firm's product at 7. If firm I advertises and II does not, consumers value I's product at 7, II's product at 3, and all the other firms products at 5. A symmetric result occurs if II advertises but I does not. (Advertising by one firm changes tastes so that the outputs are not seen as perfect substitutes.)

There are four possible configurations of "occupational choices:" (NA,NA), (NA,A), (A,NA), and (A,A), where the first element refers to firm I and the second to firm II.

(a) Conditional on each of the occupational choices, find the perfectly competitive equilibrium price and the payoffs to firms I and II.

(b) What is the equilibrium occupational choice?

Define the *ex-ante* social marginal product of a firm *relative to its occupational choice* as the difference between the maximum total gains given the occupational choices and the maximal total gains when the firm withdraws from its occupation and produces nothing. (Withdrawing from NA is the same as NA.) At each occupational choice, define Pigovian taxes/subsidies as the difference between the individual's *ex-ante* social marginal product and his market gain.

(c) Compute the Pigovian taxes/subsidies for firms I and II under each of the four occupational choices.

(d) Under these Pigovian taxes/subsidies, i.e., when individuals receive their ex ante social marginal products, what are the possible equilibria? Can you explain why, even though the firms receive their social marginal products, the Pigovian scheme does not rule out inefficient equilibria.