UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2005)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Production and cost

A firm can purchase the input vector $z \in \mathbf{R}_+^m$ at a cost of $r \cdot z$ where each input price is strictly positive. The output vector $q \in \mathbf{R}_+^n$. The firms production set $(q, z) \in Y$ is closed and convex.

(a) What is the "cost function" of the firm?

(b) Show that the cost function is a concave function of the input price vector.

(c) Is the cost function also a concave function of the output vector q? (Prove any claim.)

(d) Suppose that the firm is also a price-taker in output markets. Let $q(p, r)$ be its profit-maximizing output vector. If there are only 2 outputs, what can you say about the effect on $q(p, r)$ as p_1 rises. (You must prove any claim.)

(e) A firm has a production function $q = \ln(1 + z_1 + z_2)$. Solve for the firms cost function.

2. Choice over Time

An individual lives for T periods. He has an initial capital stock of K_1 and no other source of income. The interest rate is r. Therefore if he consumes c_t in period t the capital next period is $K_{t+1} = (1 + r)(K_t - c_t)$. The consumer has lifetime utility $U(c) = \sum_{t=1}^{T} \delta^{t-1} u(c_t)$, where $u'(c) = 1/c^{1/\sigma}$ and $\sigma > 0$. You should assume throughout that $1 + r > (1 + r)\delta > 1$.

(a) Write down the optimization problem with all T capital accumulation constraints and hence show that

$$(i)\ \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1+r} \qquad \text{and} \qquad (ii)\ \frac{c_{t+1}}{c_t} = (1+r)\theta, \quad \text{where } \theta = \frac{((1+r)\delta)^{\sigma}}{1+r}$$

(b) Explain why $\theta > 1$ if σ is sufficiently large and $\theta < 1$ if σ is sufficiently close to zero.

(c) Sketch the phase diagram for a long path.

(d) Show that the capital constraints can be written as a single life-cycle constraint. Then appeal to (ii) and solve to show that

$$c_1(T) = \frac{1 - \theta}{1 - \theta^T} K_1.$$

(e) Examine the solution as T grows large if (a) $\theta < 1$ and (b) $\theta > 1$.

(f) Comment on the solution to the infinite horizon problem is each case.

3. Hunter-Gatherer

Two players must decide whether to be hunters or gathers. If both are hunters, both receive 0; if both are gatherers both receive 1. If one is a hunter and one a gatherer, the hunter receives 3 and the gatherer 2.

(a) Find the normal form of this game.

(b) Find the Nash equilibrium of this game.

(c) Are there any dominated strategies?

(d) Find the pure and mixed Stackelberg equilibrium in which player 1 moves first.

(e) Find the minmax for both players.

Now suppose that the game is infinitely repeated:

(f) Player 1 is a long-run player with discount factor δ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor.

(g) Find strategies that support the best equilibrium from part (f).

(h) Player 1 and 2 are both long-run players with common discount factor δ. When δ is close to one describe the set of perfect equilibrium payoffs to both players.

(i) Find a discount factor and strategies for part (h) such that both players receive an equilibrium payoff of 2.5.

4. Auto Repair

A long-lived auto repair shop with discount factor $\delta > 0$ faces a sequence of short-lived car owners. The car owners must each decide whether to have their cars repaired or not. If they do, the repair shop must decide whether to repair the car or not. If the car is not repaired, the probability it will work is $1 > \pi > 0$. If it is repaired, the probability it will work is $\theta \in (\pi, 1]$. The price of the repair is $p > 0$; the cost of repair to the shop is $0 < c < p$. A car that does not work is worth nothing. A car that works is worth ν. Assume that $(\theta - \pi)\nu > p$. Car owners can only observe whether or not the car works, not whether or not the shop repaired it. In all that follows, equilibrium means perfect public equilibrium of the infinitely repeated game with public randomization.

(a) Find the extensive and normal forms of the stage-game.

(b) For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs.

(c) Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player.

(d) Find the best equilibrium for the repair shop as a function of the parameters.

5. Ordinal versus Cardinal Interpretations of Efficiency

Preferences $\succeq$ are defined on divisible consumption $c \geq 0$ and indivisible employment, 0 (unemployed) or 1 (employed). More consumption is always better than less and for any c, $(c, 0) \succ (c, 1)$ (employment is a bad). Define $\Delta(c)$ as

$$(c, 0) \sim (c + \Delta(c), 1),$$

the added consumption to be indifferent between working or not working while consuming c.

(a) Using the connection between indifference and the description of preferences, show via a diagram that $\Delta(c)$ defines the preference relation.

There is a mass 2 of identical individuals with $\Delta(c) = (1 + c)$. Besides their capacity to supply labor in the amount 1 (or 0), each individual has 1 unit of inelastically supplied capital. Technology for producing the consumption good is given by the aggregate function $2^{3/2} L^{1/2} K^{1/2}$.

(b) Show that for the above economy, $p = 1$ (price of consumption), $w = 2$ (price of labor), and $p_K = 1$ are market-clearing prices with half the population choosing to be unemployed.

(c) Suppose fractional employment were allowed. Assume points of indifference are convexified to extend preferences and the individual's budget set expands to $\{(c, b) : pc = p_K \cdot 1 + wb, b \in [0, 1]\}$. Show, via a diagram, that the equilibrium in (b) would also be an equilibrium with fractional employment.

(d) Let $u(c) \equiv u(c, 0)$ be the utility of consuming c without working and $u(c, 1) = u(c) - e$ be the utility of consuming c when employed. Show that if $u(c) = \ln(1 + c)$ and $e = \ln 2$, then $\Delta(c) = (1 + c)$.

FACT: If everyone agrees to a 50-50 chance of employment so that half the population is always employed and the aggregate output is shared equally, individual consumption would be $2^{3/2}(1)^{1/2}2^{1/2}/2 = 2$ and individual expected utility would be $\frac{1}{2}\ln(1+2) + \frac{1}{2}(\ln(1+2) - \ln 2) = \ln 3 - \frac{1}{2}\ln 2$. This is greater than individual utility in (b), e.g., the unemployed in (b) having wealth $p_k \cdot 1 = 1$ consume one unit of c implying utility $u(1) = \ln 2 < \ln 3 - \frac{1}{2}\ln 2$.

(e) Because the allocation in (b) is a Walrasian equilibrium, it is known to be Pareto optimal. Doesn't the above FACT exhibiting Pareto-superior expected utility contradict that claim? Explain. (Suggestion: In (c), how does $(c, b) = (2, 1/2)$ compare with $(1, 0)$ and $(3, 1)$? How do they compare in the above FACT when $b = 1/2$ is interpreted as a 50-50 chance of having to work while always consuming 2?)

6. Public Versus Private Goods

In one setting, choices are PUB $= \{0, 1\}$, e.g., the color of the asphalt on streets in Los Angeles will be black (0) or green (1). In another setting, the choices are PRI $= \{s_0, s_1, \ldots, s_m\}$, where s_i means that i, $i = 1, \ldots, n$ gets a two-week (expenses paid) trip to Tokyo and everyone else stays home, and s_0 means everyone stays home. In either setting, i's tastes for the outcome s is determined by the number v_i according to:

- $v_i(s, \mathbf{v}) = v_i$, if $s = 1$ and $v_i(s, \mathbf{v}) = 0$ if $s = 0$, when $s \in$ PUB
- $v_i(s, \mathbf{v}) = v_i$, if $s = s_i$, and $v_i(s, \mathbf{v}) = 0$ otherwise, when $s \in$ PRI,

where $\mathbf{v} = (v_1, \ldots, v_n)$ describes the tastes of individuals whether the choices are in PUB or PRI. Assume $v_i \in [-1, 1]$. There are no cost differentials between choices in PUB or among those in PRI, so treat costs as zero.

(a) What are the conditions for an optimal choice for $\mathbf{v}$ in PUB? In PRI? Explain the difference.

A *mechanism* is a mapping from economies $\mathbf{v}$ to $(s(\mathbf{v}), p_1(\mathbf{v}), \ldots, p_n(\mathbf{v}))$, where $s(\mathbf{v}) \in$ PUB or PRI and $p_i(\mathbf{v})$ it the money payment made by or to i. The utility to i when his tastes are v_i is $v_i - p_i$.

(b) Define the conditions for a mechanism to encourage individuals to accurately reveal their preferences. Is there a different condition for PUB than for PRI?

(c) Suppose the mechanisms for PUB and PRI satisfy the optimality condition in part (a). Give sufficient conditions under which the mechanisms would also satisfy the incentive compatibility condition in part (b). Briefly, outline an argument to justify your claim. Do the conditions and the demonstration of optimality and incentive compatibility vary between PUB and PRI?

Two conditions on a mechanism are: *weak budget balancing*, i.e., for all $\mathbf{v}$, $\sum_i p_i(\mathbf{v}) \geq 0$ (money payments not less than cost); and, *voluntarism*, i.e., for all $\mathbf{v}$, $v_i(s(\mathbf{v}), \mathbf{v}) - p_i(\mathbf{v}) \geq 0$ (utility not less than *status quo*).

(d) Do the conditions for an optimal incentive compatible mechanism in (c) imply differences with respect to weak budget balancing and voluntarism for PUB compared to PRI? Explain.

(e) The above comparisons between PUB and PRI are based on a fixed, finite number of individuals, n. Suppose n were very large. How would that change your answer to (d)?