

UCLA
Department of Economics
Ph. D. Preliminary Exam
Micro-Economic Theory
(FALL 2003)

Instructions:

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Firm scale

A firm with production function $q = F(z)$ is a price taker in all input markets. The input price vector is r .

(a) Show that the marginal cost of the firm will rise with an increase in the j^{th} input price if and only if the j^{th} input is normal.

(b) Show that $\frac{\frac{\partial}{\partial r_j} MC(q)}{\frac{\partial}{\partial r_j} AC(q)} = E(z_j, q)$, where $E(z_j, q)$ is the output elasticity of input demand.

Henceforth suppose that the firm is in an industry of identical firms with U-shaped cost curves. Entry into the industry is free.

(c) Under what conditions will the output of each active firm rise when the price of the j th input rises?

(d) Suppose that the production function is $F(z) = \begin{cases} 0 & \text{if } z_1 < \gamma, \\ (z_1 - \gamma)^\alpha z_2^\beta & \text{if } z_1 \geq \gamma \end{cases}$, where $\alpha, \beta, \gamma > 0$, $\alpha + \beta < 1$. For which of the inputs will the output of an active firm rise when the input price increases.

Note: You may make use of the fact that at the minimum AC output, $k_1 E(z_1, q) + k_2 E(z_2, q) = 1$, where k_i is the cost share of input $i = 1, 2$.

2. The Big Game

There are equal numbers of Bruin and Trojan fans. The average wealth of a Bruin fan is W_1 and the average wealth of a Trojan fan is W_2 . Bruins fans believe that their football team will win with probability 0.8. Trojan fans believe that the Bruins will win with probability 0.4. Each fan has a constant relative risk aversion of 1. The new Governor has made betting on football legal.

(a) Confirm that $v(c) = \ln c$ is a VNM utility function for each fan.

(b) If total Trojan wealth is equal to total Bruin wealth the equilibrium odds will be 3 : 2 in favor of the Bruins. All fans will bet over one quarter of their wealth. True or False. Derive your result.

(c) If the total wealth of the Bruins fans is higher, the market odds will rise to somewhere between 3 : 2 and 4 : 1. True or False? Explain carefully.

(d) Discuss the effect on the market odds if Bruin fans become more risk averse.

3. Heterogeneous Self Confirming Equilibrium

Consider the following “centipede” game. Three players move in turn. The first two players may either “drop” ending the game, or “pass” the move the next player. The third player also may either drop or pass, but the game ends either way. If player 1 drops the payoffs are (5, 3, 5). If player 2 drops the payoffs are (4, 5, 4). If player 3 drops the payoffs are (3, 4, 3). Finally, if all players pass, the payoffs are (8, 6, 8).

Show that there is no Nash equilibrium in which (pass, drop, *) occurs with positive probability. Show that Player 1 passing, player 2 randomizing 50 – 50 and player 3 passing is a heterogeneous self-confirming equilibrium. What does this imply about the connection between public randomizations over Nash equilibrium and heterogeneous self-confirming equilibrium?

4. Peasant Dictator

A peasant may eat the seed corn, yielding 1 to himself and 0 to the dictator. Or he may plant the seedcorn. If he plants, the dictator may seize the seedcorn, yielding 3 to himself and nothing to the peasant, or he may tax it lightly, yielding 1 to himself and 2 to the peasant. Show that in any Nash equilibrium the peasant eats the corn. Find a critical discount factor so that if the game is infinitely repeated between a long-run dictator whose patience exceeds the critical level and short-run peasant, there is a subgame perfect equilibrium in which the peasant grows the corn and the dictator taxes it lightly. Describe the equilibrium strategies.

5. Representative individual

Preferences are represented by the utility function

$$u(x, y) = \max \{ \min \{ x, 4y \}, \min \{ 4x, y \} \}$$

and the initial endowment is $(\alpha, \beta) \geq 0$.

- (a) As a helpful preliminary, draw an accurate diagram of the indifference curves for u .
- (b) With one individual, are there any values of (α, β) for which there is a price-taking (i.e., no trade) equilibrium. Explain.
- (c) Is $(\alpha, \beta) = (2, 3)$ such a value? Demonstrate.

- (d) Suppose several identical individuals with $(\alpha, \beta) = (2, 3)$. Show that they could gain by trading. What is the minimum number of such individuals that could exhaust all the mutual gains?
- (e) With a large population of identical individuals demonstrate why there is a price-taking equilibrium for any $(\alpha, \beta) \geq 0$.
- (f) If identical individuals with characteristics $(u, (\alpha, \beta))$ gain by trading among themselves, it would be incorrect to represent their behavior as identical. Nevertheless, show that there is a way to summarize the behavior of many $(u, (\alpha, \beta))$ by a single “representative” individual with characteristics $(U, (\alpha, \beta))$. What is U ?

6. Regulating a monopolist with unknown cost

A public utility has constant marginal cost $c \in [0, 1]$ and no fixed costs faces a demand for its output known to be $D(p) = 1 - p$. A regulatory agency wants an efficient quantity to be produced whatever the value of c , which is known only to the utility. Call $q(c')$ the efficient quantity of output when the reported cost is c' .

- (a) Find $q(c')$.
- (b) Given that $q(c')$ will be implemented, suppose the regulator adopts a “normal profits” scheme which promises to cover costs by giving the utility the total revenue $q(c') \times c'$ when it reports c' . Is there any true c for which it is incentive compatible to report truthfully? E.g., is $c = 1/2$ one of them?
- (c) Given that the regulator will implement $q(c')$, find a scheme that would encourage the utility to report its costs truthfully, no matter what they are.
- (d) Suppose the true $c = 1/2$. Demonstrate why, with respect to the scheme in (c), it will be in the interests of the utility to report $1/2$ rather than any other $c' \in [0, 1]$.