Department of Economics **Econometrics Field Exam**
UCLA **Spring, 2011**

Second Year Field Examination in Econometrics June 22, 2011

There is a total of six questions, two in each part of the exam. Please answer four out of the six available questions and at least one question from each of the first two parts, i.e., Part I and Part II.

Please provide as detailed information as possible. please keep in mind that the answers need not be long for them to be precise.

Part I:

Question 1

Suppose that you are interested in estimating the functions m and s and the distribution of $(\varepsilon_1, \varepsilon_2)$ in the following model:

$$\begin{aligned} y_1^* &= z + m\left(y_2, \varepsilon_1\right), \\ y_2 &= s\left(x, \varepsilon_2\right), \end{aligned}$$

$$y_1 = \begin{cases} 1 & if \quad y_1^* \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

where y_1, y_2, x, and z are observable random variables, $(\varepsilon_1, \varepsilon_2)$ and y_1^* are unobservable, the functions m and s are differentiable, and the distribution of $(\varepsilon_1, \varepsilon_2)$ is also differentiable.

Assume that (i) z, x, and $(\varepsilon_1, \varepsilon_2)$ are mutually independent, (ii) m is strictly increasing in ε_1, (iii) s is strictly increasing in ε_2, (iv) the support of z is R, (v) the support of x is R, and (vi) the support of $(\varepsilon_1, \varepsilon_2)$ is R^2.

1. Imposing additional conditions when necessary, obtain identification results for the function s, the function m, and the distribution of $(\varepsilon_1, \varepsilon_2)$.
2. Justify the additional conditions that you used in (1).
3. Describe a nonparametric method to estimate the functions s and m, under the conditions that you imposed in (1) and additional ones if necessary.
4. Describe the asymptotic properties of the estimator that you provided in (3).

Question 2

Consider the following model

$$\begin{aligned} y_1 &= m^1\left(y_2, \beta_1' x_1 + \varepsilon_1\right), \\ y_2 &= m^2\left(y_1, \beta_2' x_2 + \varepsilon_2\right), \end{aligned}$$

where the functions m^1 and m^2 are differentiable unknown functions, each strictly increasing in its last coordinate, (x_1, x_2) is distributed independently of $(\varepsilon_1, \varepsilon_2)$, and the support of $(\varepsilon_1, \varepsilon_2)$ is R^2.

1. Imposing additional conditions as necessary, provide conditions under which β_1 and β_2 are identified.

2. Imposing even more additional conditions as necessary, provide conditions under which the derivatives of m^1 and m^2 are identified.

3. Provide estimators for β_1 and β_2 and discuss their asymptotic properties.

4. Provide estimators for the derivatives of m^1 and m^2 and discuss their asymptotic properties, under additional assumptions if necessary.

Part II:

Question 1

Consider the model

$$y = x\beta + \varepsilon,$$

where we know $E[\varepsilon| z] = 0$. We use the following two-step estimator of β:

1. Estimate $E[x| z]$ by nonparametric regression $\widehat{E}[x| z]$.

2. Compute the OLS of y on x and $\widehat{v} = x - \widehat{E}[x| z]$.

3. The estimator $\widehat{\beta}$ of β is the OLS coefficient of x in the second step. Assuming that $\widehat{\beta}$ is consistent, find the asymptotic variance of $\widehat{\beta}$ using Newey's (1994) formula.

Question 2

Adopt the following notations:

- For each individual, there are four potential values Y_0, Y_1 and $D(0), D(1)$. We also observe an instrument Z, which takes two values, $\underline{z}$ and $\overline{z}$.

- Y_1 denotes the potential outcome under treatment, and Y_0 denotes the potential outcome under control.

- Z takes two possible values, $\underline{z}$ and $\overline{z}$.

- $D(\overline{z})$ denotes the potential treatment when $Z = \overline{z}$, and $D(\underline{z})$ denotes the potential treatment when $Z = \underline{z}$.

- We observe Z, $D = D(Z)$, and $D(Z)Y_1 + (1 - D(Z))Y_0$

- We assume that $(Y_0, Y_1, D(\underline{z}), D(\overline{z}))$ is independent of Z

- We assume that $D(\underline{z}) \leq D(\overline{z})$.

1. Prove that

$$E[Y_1 - Y_0| D(\overline{z}) - D(\underline{z}) = 1] = \frac{E[Y| Z = \overline{z}] - E[Y| Z = \underline{z}]}{E[D| Z = \overline{z}] - E[D| Z = \underline{z}]}.$$

2. Now, suppose that

$$\lim_{\overline{z}\to\infty} \Pr\left[D\left(\overline{z}\right)=1\right|\overline{z}\right]=1,$$
$$\lim_{\underline{z}\to-\infty} \Pr\left[D\left(\underline{z}\right)=0\right|\underline{z}\right]=1.$$

Prove that
$$E\left[Y_1-Y_0\right]=\lim_{\overline{z}\to\infty,\underline{z}\to-\infty}\frac{E\left[Y\right|Z=\overline{z}\right]-E\left[Y\right|Z=\underline{z}\right]}{E\left[D\right|Z=\overline{z}\right]-E\left[D\right|Z=\underline{z}\right]}.$$

Part III:

Question 1—Dynamic Programming:

Consider a finite horizon discrete dynamic programing model in which an individual makes decisions about three variables: consumption c_t, leisure l_t, and investment in health I_t^h. An individual may decide not to work, in which case he/she will consume the full amount of available leisure.

Each period utility is given by

$$u_t\left(c_t,l_t,h_t\right)=u\left(c_t,l_t,h_t\right)=K_0c_t^{\gamma_1}l_t^{\gamma_2}h_t^{\gamma_3}.$$

Each period an individual gets an amount of unearned income a_t. If in addition, the individual works, then he/she is paid an hourly wage w_t, which is a function of the number of years of education denoted e, and experience, denoted x. Specifically, we have

$$\log w_t=\alpha_0+\alpha_1e_t+\alpha_2e_t^2+\alpha_3x_t+\alpha_4x_t^2+\alpha_5x_te_t+\varepsilon_t,$$

where ε_t is an idiosyncractic shock, uncorrelated with either education or experience. The current level of health is given by
$$h_t=\exp\left\{\delta_0+\delta_1h_{t-1}+\delta_2I_t^h+\nu_t\right\}.$$

The individual starts with an endowment of B_0, and earns an interest rate of r_t for any unused monetary resources carried over from period t to period $t+1$. Assume that the interest rates for the three periods are known in advance.

For the questions specified below, if you think that some necessary information has been omitted, please make assumptions about that information.

1. Define the state vector, say z_t.

2. Specify all the necessary budget constraints, regarding money and time.

3. Specify the Bellman representation of the value function. Make sure to provide the value function for *all* possible states of nature and choices.

4. Provide the full list of parameters that need to be estimated. Let the true parameter vector be denoted by θ_0.

5. Provide details about the data that one would need in order to be able to estimate θ_0.

6. Provide detailed information about the method by which you propose to estimate the parameter vector θ_0. You should answer this question providing enough detail so that a professional programmer who knows nothing about economics would be able to program the procedure. (Keep in mind that there are several alternative ways for obtaining an estimate for θ_0.)

7. What is the asymptotic distribution for the parameter vector estimate, say $\widehat{\theta}_n$, obtained following the procedure suggested in part (6)? Provide brief justifications for all your claims.

Question 2—Generalized Method of Moments:

Consider the two moment functions given by $\varphi_1(W_i, \theta_1)$ and $\varphi_2(W_i, \theta_1, \theta_2)$. Suppose that, when evaluated at the true population parameter vectors, θ_{01} and θ_{02}, respectively, we have

$$\begin{aligned} E\left[\varphi_1(W_i, \theta_{01})\right] &= 0 \quad \text{and} \\ E\left[\varphi_2(W_i, \theta_{01}, \theta_{02})\right] &= 0, \end{aligned}$$

where W_i, $i = 1, ..., n$, represent cross-section independent and identically distributed (i.i.d.) data.

Let $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ be $M_1 \times 1$ and $M_2 \times 1$ vector-valued functions, respectively, and correspondingly let θ_{01} and θ_{02} be d_1 and d_2 vectors of parameters, with $M_1 > d_1$ and $M_2 = d_2$.

Assume that you have already obtained an estimator for θ_{01}, say $\widehat{\theta}_{n1}$, and this estimator is consistent and asymptotically normal, i.e.,

$$\begin{aligned} &\widehat{\theta}_{n1} \xrightarrow{p} \theta_{01}, \\ &\sqrt{n}\left(\widehat{\theta}_{n1} - \theta_{01}\right) \xrightarrow{D} N\left(0, V_1\right), \end{aligned}$$

as $n \to \infty$.

1. Suppose that it has been determined that θ_{01} has to be computed using the simulated method of moments (SMM) method. What would be the asymptotic distribution of the estimator for θ_{01}, say $\widehat{\theta}_{n1}$? Justify each and every step in your answer.

2. Suggest an efficient Generalized Method of Moments (GMM) estimator for θ_{02}, using a plug-in estimator for θ_{01}, namely $\widehat{\theta}_{n1}$.

3. Show that the estimator suggested in (2) is consistent and has an asymptotic normal distribution. In doing so explicitly state all the assumptions you make. Provide the asymptotic properties of the estimator $\widehat{\theta}_{n2}$ for θ_{02}. Justify each and every step in your answer.

4. Suppose now that one would like to develop a test for the null hypothesis H_0: $h_1\left(\theta_{01}\right) - h_2\left(\theta_{02}\right) = 0$. Suggest a test and give details regarding the exact procedure you would follow. Be sure to give enough details so one can apply the procedure to a specific data.