

**UCLA Department of Economics**  
**Second-Year Field Examination in Econometrics**  
**Spring 2007**

**Instructions:**

Solve all three Parts I-III.

Use a separate bluebook for each part.

You have 4 hours to complete the exam.

Calculators and other electronic devices are not allowed.

**GOOD LUCK!!!**

PART I (Based on Kyriazidou's course)

**Problem 1:**

(a) Suppose that

$$y_{it} = 1 \{x_{it}\beta + \alpha_i - \varepsilon_{it} \geq 0\} \quad i = 1, \dots, N; t = 1, 2$$

where  $\varepsilon_{it}$  are unobservable variables distributed independently and identically over time conditional on  $(x_{i1}, x_{i2}, \alpha_i)$ ,  $x_{it}$  is a  $1 \times k$  vector of observable variables, and  $\alpha_i$  is an unobservable individual-specific effect. Discuss identification and estimation of  $\beta$ . Assume that cross-sectional sampling is random. Make sure to mention other important assumptions that are used in the identification and consistent estimation of  $\beta$ .

(b) Now suppose that

$$y_{it} = 1 \{\beta y_{it-1} + \alpha_i - \varepsilon_{it} \geq 0\} \quad i = 1, \dots, N; t = 1, 2, 3$$

where  $\varepsilon_{it}$  are distributed independently and identically over time conditional on  $\alpha_i$  and  $\alpha_i$  is an unobservable individual-specific effect. Assume that  $y_{i0}$  is observed for each  $i$  although it is not necessarily generated by the same model as the subsequent  $y_{it}$ 's. Discuss identification and estimation of  $\beta$ . Assume that cross sectional sampling is random.

**Problem 2:**

(a) Describe how you would perform Chamberlain's strict exogeneity test in a linear static panel data model of the form

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it}$$

Make sure to explain the intuition/idea behind the test, to describe the underlying assumptions and to derive its asymptotic distribution. For simplicity you may assume that  $x_{it}$  is scalar.

(b) Discuss how you would perform the same test for the static panel data logit model of the form

$$y_{it} = 1 \{x_{it}\beta + \alpha_i + \varepsilon_{it} \geq 0\}$$

(HINT: The strict exogeneity concept need to be strengthened from linear projection to conditional mean independence.)

## PART II (Based on Winkelmann's course)

### Problem 1:

Santos Silva and Tenreyro argue in their 2006 REStat paper that “the Poisson PML estimator has the essential characteristics needed to make it the new workhorse for the estimation of constant-elasticity models.” (p. 649) Explain the reasoning behind their argument.

### Problem 2:

Consider the linear exponential family (lef) of distributions with density function

$$f(y; m) = \exp\{A(m) + B(y) + C(m)y\}$$

where  $A$  and  $C$  are twice continuously differentiable functions and  $m = E(y)$ .

- Show that the Poisson distribution is a linear exponential family.
- Show that the score function for an iid sample of size  $n$  from any lef distribution can be written as

$$\frac{\partial \log L}{\partial m} = \sum_{i=1}^n \frac{\partial C(m)}{\partial m} (y_i - m)$$

- What does the result under b) imply for the consistent estimation of the mean parameter of an otherwise misspecified model?

### Problem 3:

Briefly describe the consequences of unobserved heterogeneity in

- the Poisson regression model
- the proportional hazards model

(no formal derivations!)

### Problem 4:

In a study of unemployment durations an exponential model is used to analyze the data. To be specific, the rate at which the unemployed leave unemployment is specified as  $\exp(x'\beta)$  with  $x$  a vector of explanatory variables.

- What is the density (pdf) of a complete unemployment spell  $t$ ?
- What is the distribution function (cdf) of a complete unemployment spell  $t$ ?

- c) Let one of the explanatory variables be education in years. What is the average effect of a one year increase in education on the logarithm of the average duration of an unemployment spell?

Instead of observing a random sample of complete unemployment spells, you have a random sample of unemployment spells from the records of the unemployment benefits office. Because there is a waiting period of 1 month for unemployment benefits, only unemployed who stay unemployed for more than 1 month receive benefits and enter the records of the unemployment benefits office.

- d) Give the log-likelihood function  $L(\beta)$  if we follow the unemployed until they leave unemployment.
- e) How would this log-likelihood function change if we follow the unemployed for at most 6 months?
- f) (optional) Consider an alternative sampling scheme: a sample is taken from the stock of unemployed at time 0. You observe the elapsed duration  $\tau$  and the total duration  $t$ . The probability for a spell starting at time  $-\tau$  is constant:  $p(-\tau) = \alpha$ . The distribution of durations  $g(t; \theta)$  is independent of the starting time  $-\tau$ . Determine
- $f(t, \tau | t > \tau)$ , and
  - $f(t | t > \tau)$

and show that the distribution of (observed) unemployment durations in such a sample is “length biased”.

### PART III (Based on Guggenberger's course)

#### Problem 1:

True/Questionable/False? No points are given for just stating true or false, it is the explanation what counts.

1) An MA( $q$ ) process (for a finite positive integer  $q$ ) is ergodic.

2) For the Geweke Porter–Hudak estimator  $\hat{d}$  of the long memory parameter  $d$  the number of frequencies  $m$  used in the pseudo OLS regression has to go to infinity with increasing sample size to ensure that the bias of  $\hat{d}$  goes to 0.

3) If one regresses a random walk  $y_{1,t}$  on its own lagged observation  $y_{1,t-1}$ , on an independent random walk  $y_{2,t}$  and on its lagged observation  $y_{2,t-1}$  i.e.  $y_{1,t} = \hat{\alpha}y_{1,t-1} + \hat{\beta}y_{2,t} + \hat{\gamma}y_{2,t-1}$  then the theory of spurious regression implies that the OLS estimator of  $\beta$  is inconsistent.

#### Problem 2:

1) In the linear iid IV model

$$\begin{aligned}y_i &= x_i\theta + u_i, \\x_i &= z_i\pi + v_i,\end{aligned}\tag{1}$$

derive the asymptotic distribution of the t statistic (that tests  $H_0 : \theta = 0$  versus a two sided alternative) under weak instrument asymptotics  $\pi = n^{-1/2}c$  for a fixed constant  $c$ . Assume that both  $\theta$  and  $\pi$  are scalars and that the errors are conditionally homoskedastic.

2) Explain the intuition and implementation of Moreira's (Ecta, 2003) conditional likelihood ratio test for the test  $H_0 : \theta = 0$  versus a two sided alternative in model (1).

#### Problem 3:

The goal is to test  $H_0 : \theta = \theta_0$  against  $H_1 : \theta > \theta_0$  for given iid data  $X_1, \dots, X_n$  with  $EX_i = \theta$ . The test for  $H_0$  is to reject if  $T_n = (\hat{\theta} - \theta_0)/s(\hat{\theta}) > c$ , where  $c$  is picked so that the type I error is  $\alpha$  ( $\hat{\theta}$  is a root- $n$  consistent estimator of  $\theta$  and  $s(\hat{\theta})$  is a consistent estimator of the standard deviation of  $\hat{\theta}$ ). Compare the following two approaches to do so:

1) Using the non-parametric bootstrap, you generate  $B$  bootstrap samples, calculate  $\hat{\theta}^*, s(\hat{\theta}^*)$  for each resample and then calculate  $T_n^* := (\hat{\theta}^* - \theta_0)/s(\hat{\theta}^*)$ ,  $B$  times. Let  $q_n^*(1-\alpha)$  denote the 100(1- $\alpha$ )% quantile of the empirical distribution of  $T_n^*$ . You reject  $H_0$  if and only if  $T_n > q_n^*(1-\alpha)$ .

2) Using subsampling, you calculate  $T_{b,i} = (\hat{\theta}_{b,i} - \theta_0)/s(\hat{\theta}_{b,i})$ , where  $\hat{\theta}_{b,i}$  and  $s(\hat{\theta}_{b,i})$  are based on the same formula as  $\hat{\theta}$  and  $s(\hat{\theta})$  but instead of using all the data, they only use the data  $\{X_i, \dots, X_{i+b-1}\}$  for  $i = 1, \dots, n - b + 1$ . Here  $b$  is the blocksize that satisfies  $b/n \rightarrow 0$ . Let  $q_{n,b}(1-\alpha)$  denote the 100(1- $\alpha$ )% quantile of the empirical distribution of  $T_{b,i}$  for  $i = 1, \dots, n - b + 1$ . You reject  $H_0$  if and only if  $T_n > q_{n,b}(1-\alpha)$ .

Discuss the power properties of the two tests. Are they consistent?

**Problem 4:**

Explain briefly how the Phillips and Perron unit root test works in the regression model

$$y_t = \rho y_{t-1} + u_t,$$

where the error term  $u_t$  is possibly serially correlated and heteroskedastic.