Spring 2006 UCLA Department of Economics
Field Examination in Econometrics

Instructions:
Choose and Solve Three Parts in Parts I – IV
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

# 1 Part I

**Answer every question.**

1. Suppose that $(D_i, Y_{1i}, Y_{0i})$ $i = 1, \ldots, n$ are iid, where $D_i$ denotes the binary variable that indicates treatment status. We only observe $(D_i, Y_i)$ where $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$. Let $n_1$ and $n_0$ denote the sizes of subsamples consisting of treated and controls. Let $n = n_1 + n_0$. Compare the probability limit of

$$\frac{\sum_{D_i=1} Y_i}{n_1} - \frac{\sum_{D_i=0} Y_i}{n_0}$$

in relation to the ATET

$$\gamma \equiv E\left[ Y_{1i} - Y_{0i} \mid D_i = 1 \right]$$

2. Now suppose that $D_i$ is randomly assigned, i.e.,

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i})$$

Let

$$\begin{aligned} \alpha &\equiv E[Y_{0i}], \quad \beta \equiv E[Y_{1i}] - E[Y_{0i}] \\ U_{0i} &\equiv Y_{0i} - E[Y_{0i}], \quad U_{1i} \equiv Y_{1i} - E[Y_{1i}] \end{aligned}$$

Prove that

$$Y_i = \alpha + D_i \beta + \varepsilon_i$$

where

$$\varepsilon_i = D_i U_{1i} + (1 - D_i) U_{0i}$$

Prove that

$$E[\varepsilon_i] = 0$$

Also prove that

$$E[D_i \varepsilon_i] = 0$$

Is OLS of $Y_i$ on a constant and $D_i$ going to yield a consistent estimator of $\beta$? (This is a slightly general framework than the lecture note material, where we imposed the constant treatment effects assumption.)

3. (For this question, you may want to define your own notation.) Suppose that we do not have the situation where $D_i$ is random conditional on some set of covariates $X_i$, i.e., we do NOT have

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) \mid X_i$$

Let $\beta(x) \equiv E\left[ Y_{1i} - Y_{0i} \mid X_i = x \right]$. Let

$$p(X_i) \equiv \Pr\left[ D_i = 1 \mid X_i \right]$$

denote the propensity score, which we assume is known to the econometrician. Do we necessarily have

$$\beta(x) = \frac{E[D_i Y_i | X_i = x]}{p(x)} - \frac{E[(1 - D_i) Y_i | X_i = x]}{1 - p(x)}?$$

Why or why not? Can we say that

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\widehat{E}[D_i Y_i | X_i]}{p(X_i)} - \frac{\widehat{E}[(1 - D_i) Y_i | X_i]}{1 - p(X_i)}$$

is consistent for the ATE $\beta \equiv E[Y_{1i}] - E[Y_{0i}]$? Here, $\widehat{E}[D_i Y_i | X_i]$ and $\widehat{E}[(1 - D_i) Y_i | X_i]$ denote some nonparametric estimates of $E[D_i Y_i | X_i]$ and $E[(1 - D_i) Y_i | X_i]$. Why or why not?

## 2 Part II

**Answer every question.**

1. a) Consider the following linear i.i.d. instrumental variables model

$$\begin{aligned} y_i &= Y_i\theta + u_i, \\ Y_i &= z_i\pi + v_i, \end{aligned}$$

$i = 1, ..., n$ ($n$ being the sample size), where both $\pi$ and $\theta$ are scalars. We are interested in testing the null hypothesis $H_0 : \theta = \theta_0$ versus $H_1 : \theta > \theta_0$. Derive the asymptotic null distribution of a t statistic under weak instrument asymptotics $\pi = \pi_n = c/n^{1/2}$ for a fixed number $c$. You can assume conditional homoskedasticity. What parameters does the limit distribution depend on? What is your conclusion about using a t statistic for the above test together with normal critical values?

   b) Describe two alternative testing procedures for the above null hypothesis whose null rejection probabilities are not affected by the strength or weakness of the instrument.

2. a) In the AR(1) model

$$y_0 = 0 \text{ and } y_t = \rho y_{t-1} + u_t, \; u_t = \text{iid}(0, \sigma^2)$$

($t \geq 1$) derive the asymptotic distribution of the OLS estimator $\widehat{\rho}_{OLS}$ when $\rho = 1$. If you use any theorem in your derivation, state it precisely with all assumptions needed. Explain how this result can be used as a unit root test.

   b) Explain which complications arise if we want to implement a unit root test when the errors $u_t$ are potentially serially correlated and heteroskedastic.

3. We are given data $z_1, ..., z_n$. Describe how to use subsampling to test a certain hypothesis $H_0$ against an alternative $H_1$ based on a certain test statistic $T_n = T_n(z_1, ..., z_n)$. Mention some of the key assumptions needed for subsampling to work and give the intuition behind those assumptions. What is the key difference between subsampling and the (nonparametric) bootstrap?

# 3 Part III

**Answer every question.**

1. RIGHT OR WRONG; PROVE YOUR CLAIM

   (a) The First Difference estimator is consistent in the dynamic linear panel data model

   $$y_{it} = y_{it-1}\beta + \alpha_i + \varepsilon_{it}$$

   provided that $\varepsilon_{it}$ is uncorrelated with $\varepsilon_{is}$ for all $t \neq s$, $y_{i0}$ and $\alpha_i$, and $y_{i0}$ is uncorrelated with $\alpha_i$. Consider large $N$ and small $T$ asymptotics.

   (b) The fixed effects conditional logit estimator is obtained by running a logit of the first differenced $y_{it}$ on the first differenced $x_{it}$ when $T = 2$.

2. (a) Write down a static panel data Type II Tobit model with additive individual effects.

   (b) Write down the log-likelihood of a random effects specification of the model that assumes a joint normal distribution for all unobservables. State the assumptions under which the log-likelihood is derived.

   (c) Describe Heckman's two-step method for estimating the model of part (b).

   (d) Describe a method for identifying a fixed effects specification of the model that does not require the parametrization of the distribution of the model's unobservables. Make sure to state the important assumptions that allow identification and to provide the form of a consistent estimator for the model's parameters of interest.

   (e) What are the advantages and disadvantages of the approaches in (b) and (d)?

# 4 Part IV

**Answer 2 questions.**

1. Briefly describe the difference between a trend-stationary and a difference-stationary time series. Explain how you would test which is a more accurate description of the data and discuss the consequences of incorrectly assuming trend-stationarity or difference-stationarity.

2. Explain how you would construct a test of forecast rationality for $\tau-$steps ahead forecasts when the loss function is (a) quadratic $\left(L(e_{t,\tau}) = e_{t,\tau}^2\right)$ and (b) linex $(L(e_{t,\tau}) = \exp(e_{t,\tau}) - e_{t,\tau} - 1$ What estimators of the variance would you use in constructing the test?

3. Consider the system

$$\begin{aligned} X_t &= aY_{t-1} + \varepsilon_t \\ Y_t &= bX_{t-1} + \nu_t \end{aligned}$$

where $\varepsilon_t$, $\nu_t$ are zero-mean white noise with $var(\varepsilon) = var(\nu) = 1$, $corr(\varepsilon_t, \nu_s) = 0$ if $s \neq t$, $corr(\varepsilon_t, \nu_t) = \rho$.

   (a) Under what conditions does $Y_t$ Granger-cause $X_{t+1}$?

   (b) Under what conditions are $X_t$ and $Y_t$ cointegrated in this system?

   (c) Write the error-correction representation of the system

   (d) Do your results depend on the value of $\rho$?