Choose any four questions

UCLA **Fall Semester 1999**

ECONOMETRICS

Core Exam

1. Consider inference for a scalar parameter β characterized by the moment equations:

$$E\begin{pmatrix} Z_1 - \alpha \\ Z_2 - \alpha \end{pmatrix} = 0.$$

   (a) Discuss the standard (Hansen) GMM estimator for this problem.
   (b) How would you compute the estimator?
   (c) What is the large sample distribution for the estimator?
   (d) How can you test the validity of the two moment equations?
   (e) Describe the empirical likelihood estimator for this case.

2. Consider the standard linear regression model with $E[Y|X] = X'\beta$.

   (a) What is the large sample variance of the least squareds estimator under homoskedasticity?
   (b) What is the heteroskedasticity consistent variance?
   (c) What would be a reason for using the estimator for the variance under homoskedasticity?
   (d) How can you test for heteroskedasticity using the information matrix test? Which parts of the information matrix equality can be used for testing? What are the degrees of freedom?
   (e) Describe an alternative way of testing for heteroskedasticity.
   (f) Describe how you would estimate the 0.5 quantile regression function, assuming it is a linear function of X, that is, $X'\beta_{0.5}$.
   (g) Describe two ways of estimating the asymptotic variance of $\beta_{0.5}$?

3. Discuss different approaches to inference in instrumental variables contexts with weak instruments. Include Bayesian methods, small sample bias, test for overidentifying restrictions.

1. Consider the following model:

$$y_t|y_1,\dots,y_{t-1} \sim \mathcal{N}(\beta_1 + \beta_2 y_{t-1}, \sigma^2), \qquad t = 2,\dots,T.$$

(a) Write down the likelihood function for $(y_2,\dots,y_T)$ conditional on y_1. Under what conditions will it be appropriate to carry out posterior inference based on the conditional likelihood, rather than the full joint likelihood of $(y_1,\dots,y_T)$?

(b) Assume the conventional diffuse prior for $(\beta_1, \beta_2, \sigma)$. What is the predictive distribution of y_{T+1} given $y_1,\dots,y_T$? How would you obtain 0.025 and 0.975 quantiles of this predictive distribution?

(c) In what sense is the Bayes estimator $E(\beta_2|y_1,\dots,y_T)$, that is, the posterior mean of β_2, admissible? Be precise about the appropriate loss function and the relevant repeated sampling framework.

2. Let y_i be a binary indicator for employment, and x_i a scalar regressor, for a random sample of individuals $i = 1,\dots,n$. Consider the following model:

$$Pr(y_i = 1|x_i) = \Phi(\beta_1 + \beta_2 x_i)$$

where Φ is the standard normal CDF.

(a) Using a flat (diffuse) prior for β_1 and β_2, outline a simulation-based procedure for posterior inference.

(b) Describe a method for posterior inference based on discretization. If x_i were a high-dimensional vector instead of a scalar, would this procedure be effective? Explain.

(c) Returning to the case where x_i is scalar, suppose that we modify the prior to impose the restriction that $\beta_2 > 0$. (In other words, the prior for (β_1, β_2) is flat on $(-\infty, \infty) \times (0, \infty)$). Interpret this restriction, and describe a simulation procedure for posterior inference.

I. Geweke (1996, *Journal of Econometrics*) studies the following system of equations:

$$\underset{n \times L}{\mathbf{Y}} = \underset{n \times p}{\mathbf{X}} \ \underset{p \times L}{\Theta} + \underset{n \times k}{\mathbf{Z}} \ \underset{k \times L}{A} + \underset{n \times L}{\mathbf{E}}.$$

Here, it is assumed that

$$\underset{p \times L}{\Theta} = \underset{p \times q}{\Psi} \cdot \underset{q \times L}{\Phi},$$

where $q < p$.

A. Geweke proposes two normalizations:

$$\Psi = \begin{bmatrix} I_q \\ \Psi^* \end{bmatrix}, \text{ and } \Phi = [I_q, \Phi^*].$$

Which of these normalizations is more appropriate for cointegrating Vector Error Correction Models (VECMs)? Specify the interpretation of the coefficients and the variables under this normalization. What use does Geweke propose for the other (non-VECM) normalization?

B. Suppose that

$$\Psi = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}.$$

Interpret Ψ in the VECM context. What does this structure represent? Demonstrate whether or not this structure is consistent with the VECM normalization you chose in part (A).

C. What does your answer to part (B) imply about Bayesian sensitivity to the ordering of the variables in $\mathbf{Y}$? That is, is Geweke's Bayesian approach to the analysis of VECMs affected by the ordering of the variables? Relate your answer to the results from (B).

D. Geweke proposes a particular prior for the VECM. Discuss the properties of this prior. Under this prior, which parameters, if any, are independent? Can the prior be decomposed into common, recognizable distributions? Relate Geweke's prior to the Litterman prior. Are the two consistent with one another? Explain your answer. Make sure you discuss how both priors handle unit roots, cointegration and lag terms. If the priors are consistent, show this. If they are not consistent, discuss whether or not the Geweke prior can be modified or extended to capture elements of the Litterman prior.

E. Discuss testing for cointegration within Geweke's framework. Describe

 i. the alternative models for alternative ranks of cointegration;
 ii. the basic quantities relevant for Bayesian model comparison;
 iii. how you could compute these quantities in the present context;
 iv. how this computation method compares and contrasts with Gibbs sampling.

II. You have the task of producing forecasts for the next eight quarters of prices, inventories, and unemployment. Use the following questions to describe the steps you would take to construct these forecasts, within the context of a stationary vector autoregressive model.

A. What additional series might be relevant to your objective? Contrast the definitions of Granger Non-Causality and Block Exogeneity. Are either of these constructs relevant to deciding what variables to use in your model? Explain your answer by referring to a VAR(1), that is a first order VAR model.

B. What role, if any, does model stability as defined by Lütkepohl play in constructing forecasts? What is the definition of model stability? What connection is there between stability and covariance stationarity? Describe how you would measure stability given the parameters of a VAR(2):

$$\mathbf{y}_t = B_1\mathbf{y}_{t-1} + B_2\mathbf{y}_{t-2} + \mathbf{e}_t.$$

C. What evidence regarding stability can you obtain from an analysis of the individual series? Describe alternative tests you could perform here, and when particular tests are more relevant. Specifically, what qualitative, univariate features of the data determine the appropriate test?

D. Describe two classical hypothesis testing procedures related to lag selection. Would you employ either of these testing procedures in the forecasting context? Explain. How do these procedures contrast with the Akaike Information Criterion (AIC), and the Hannan-Quinn (HQ) model selection procedure? How do the objectives of the AIC and HQ procedures differ? Is one of these procedures more relevant for your purposes?

E. How do you propose to construct confidence measures for your forecasts? Compare two, non-Bayesian methods. What are the relative strengths of each? Be as specific as possible in how you would construct confidence measures based upon these two approaches.