Fall 2004
Department of Economics
UCLA

ECONOMETRICS FIELD EXAMINATION

Instructions: This is a 4 hour closed book/closed notes exam. There are FOUR parts in the exam. Answer ALL questions of ONLY TWO parts of your choice. Use a separate bluebook for each part. GOOD LUCK!

PART I

Question 1:

Suppose that

$$y_i = G(x_i \cdot \beta) + \varepsilon_i$$

where $G(\cdot)$ is some smooth function and $\varepsilon_i \sim N(0, \sigma^2)$.

a. Show that the log likelihood is equal to

$$L\left(\beta, \sigma^2\right) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\left(\sigma^2\right) - \frac{\sum_{i=1}^n (y_i - G(x_i \cdot \beta))^2}{2\sigma^2}$$

Let

$$\sigma^2(\beta) = \underset{\sigma^2}{\operatorname{argmax}}\, L\left(\beta, \sigma^2\right)$$

Show that

$$\sigma^2(\beta) = \frac{1}{n}\sum_{i=1}^n (y_i - G(x_i \cdot \beta))^2$$

and

$$L\left(\beta, \sigma^2(\beta)\right) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\left(\frac{1}{n}\sum_{i=1}^n (y_i - G(x_i \cdot \beta))^2\right) - \frac{n}{2}$$

Conclude that the MLE for β is equal to the nonlinear least squares estimator

$$\widehat{\beta} \equiv \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^n (y_i - G(x_i \cdot \beta))^2$$

b. Let

$$\phi \equiv E[g(x_i \cdot \beta)\beta]$$

and

$$\widehat{\phi} \equiv n^{-1}\sum_{i=1}^n g\left(x_i \cdot \widehat{\beta}\right)\widehat{\beta}$$

where

$$g(s) \equiv \frac{dG(s)}{ds}$$

and $\widehat{\beta}$ denotes the MLE of β. What is the asymptotic distribution of $\sqrt{n}\left(\widehat{\phi} - \phi\right)$? How would you estimate the asymptotic variance?

Question 2:

Suppose that the binary treatment D_i is random conditional on some set of covariates X_i:

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) | X_i$$

a. Show that

$$\beta = E[\beta(X_i)]$$

where

$$\begin{aligned} \beta &\equiv E[E[Y_{1i} - Y_{0i} | X_i]] \\ \beta(x) &\equiv E[Y_{1i} - Y_{0i} | X_i = x] \end{aligned}$$

b. Show that

$$\gamma = \frac{E[\beta(X_i) \cdot p(X_i)]}{E[D_i]}$$

where

$$\gamma \equiv E[Y_{1i} - Y_{0i} | D_i = 1]$$

and

$$p(X_i) \equiv \Pr[D_i = 1 | X_i]$$

c. Show that

$$\beta(x) = \frac{E[D_i Y_i | X_i = x]}{E[D_i | X_i = x]} - \frac{E[(1 - D_i) Y_i | X_i = x]}{E[1 - D_i | X_i = x]}$$

d. Suggest an estimator of γ assuming that you have nonparametric estimators $\widehat{\beta}(X_i)$ and $\widehat{p}(X_i)$ of $\beta(X_i)$ and $p(X_i)$.

PART II

Question 1—Dynamic Programming:

Consider a three-period model in which an individual makes decisions about three variables: consumption c_t, leisure l_t, and the amount to invest in health h_t, for $t = 1, 2, 3$. Assume that all individuals are old enough, so that they cannot choose to attend school, and hence cannot increase their education levels. For simplicity assume that l_t can take on only three values: (i) l^{FT}, the amount of leisure if the person works at a *full-time* job; (ii) l^{PT}, the amount of leisure if the person works at a *part-time* job; and (iii) l_0, the amount of leisure if the person *does not work.*
Each period utility is given by

$$u_t(c_t, l_t) = u(c_t, l_t) = \kappa c_t^{\gamma_1} l_t^{\gamma_2} - \gamma_3 dh_{it}.$$

where $dh_{it} = 1$ if the person is in bad health and $dh_{it} = 0$, otherwise. The probability that a person will experience a bad health condition is an increasing function of the individual's age and a decreasing function of the amount invested in health in the proceeding period, say $h_{i,t-1}$. Denote this function by $f_h(age, h|dh = 0)$. Furthermore, assume that a bad health condition is an absorbing state, that is, once a person is in bad health he/she will stay in that state for the remainder of his/her life.
The wage function is given by

$$\log w_{it} = \alpha_i + \alpha_1 ed_{it} + \alpha_2 ed_{it}^2 + \alpha_3 ex_{it} + \alpha_4 ex_{it}^2 + \varepsilon_{it},$$

where ed denotes the education level of the individual and ex denotes his/her labor market experience.
Each individual also gets a stream of unearned income denoted by i_t.
The individual starts with an endowment of A_0, and earned an interest rate of r_t for any unused monetary resources carried over from period t to period $t+1$. Assume that the interest rates for the three periods are constant and known, say r_0.
For the questions specified below, if you think that some necessary information has been omitted, please make assumption(s) about that information.

1. Define the state vector, say z_{it}.
2. Specify all the necessary budget constraints, regarding money and time.
3. Specify the individual's value function, $V(z_{it}, t)$ for each period, $t = 1, 2, 3$.
4. Provide the full list of parameters that need to be estimated. Let the true parameter vector be denoted by ϕ_0.
5. Suppose that you are presented with unlimited resources for collecting data in order to estimate the above model. Specify in detail all the data that you will be collecting.

6. Suppose that you want to use the method of simulated moments for estimating the parameter vector ϕ_0. Provide a detailed answer as to the steps that you will need to take. You should answer this question as if you were giving instructions to a professional programmer that needs to write the program for you.

7. Provide the asymptotic distribution for the parameter vector estimate, say $\widehat{\phi}$, obtained in (6). Provide brief justifications for all your claims.

Question 2—Bootstrapping:

Consider the non-linear regression model given by

$$y_i = g\left(x_i'\beta_0\right) + \varepsilon_i,$$

where ε_i are independent, and

$$\varepsilon_i | x_i \sim \left(0, \sigma_\varepsilon^2\left(x_i\right)\right)$$

for $i = 1, ..., n$. That is, the variance of ε_i, conditional on x_i, is a function of x_i.
Suppose that you obtain an estimate for β_0, say $\widehat{\beta}$, by a non-linear least-squares method, that is,

$$\widehat{\beta} \equiv \arg\min_{\beta} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - g\left(x_i'\beta\right)\right)^2.$$

1. Suppose that you want to obtain the standard errors for $\widehat{\beta}$ by using the bootstrap method. Determine whether or not the bootstrap method is valid in this case.

2. Suppose the bootstrap method is valid, how would you obtain an estimate for the standard error vector, say se_β.

3. Suppose now that you want to construct a confidence interval for $h\left(\beta_0\right)$, where $h\left(\cdot\right)$ is some non-linear function of the parameter vector β_0. How would you obtain such a confidence interval using the information you already obtained in (2)?

4. What alternative method can you suggest for providing a confidence interval for $h\left(\beta_0\right)$? Be very specific in your answer.

5. Which of the two methods, the one in (3), or the one in (4) would you prefer. Justify your answer.

PART III

Question 1:

Consider the dynamic linear panel data model

$$y_{it} = \beta_0 y_{it-1} + \alpha_i + \varepsilon_{it}$$

where $i = 1, ..., N$ and $t = 1, ..., T$ with T of smaller order than N.
(a) Describe and justify an efficient estimator of β_0 under the assumption

$$E^* \left(\varepsilon_{it} | y_{i0}, y_{i1}, ..., y_{it-1}, \alpha_i\right) = 0$$

Here E^* denotes the best linear predictor.
(b) Assume that $y_{it}|y_{it-1}, ..., y_{i0}, \alpha_i \sim N\left(\beta_0 y_{it-1} + \alpha_i, \sigma^2\right)$. Find the density of $(y_{i1}, ..., y_{iT})$ given (y_{i0}, α_i). Is it a good idea to use the log of this density summed over i to estimate β_0 and σ^2 along with the "fixed effects" α_i? Prove your claim.
(c) If $\alpha_i | y_{i0} \sim N\left(\gamma_0 + \gamma_1 y_{i0}, \sigma_c^2\right)$ where $\sigma_c^2 \equiv Var\left(c_i\right)$ and $c_i \equiv \alpha_i - \gamma_0 - \gamma_1 y_{i0}$, write down the density of $(y_{i1}, ..., y_{iT})$ given y_{i0}. How would you estimate all the unknown parameters of the model?

Question 2:

Consider the doubly censored regression model:

$$\begin{aligned} y_i^* &= x_i \beta_0 + \varepsilon_i \\ y_i &= c_1 \text{ if } y_i^* \leq c_1 \\ &= y_i^* \text{ if } c_1 < y_i^* < c_2 \\ &= c_2 \text{ if } y_i^* \geq c_2 \end{aligned}$$

where x_i is $1 \times K$, β_0 is $K \times 1$, and $c_1 < c_2$ are known censoring points.
(a) Describe the ML estimator of the unknown parameters of the model under the assumption that $\varepsilon_i | x_i \sim N\left(0, \sigma^2\right)$.
(b) Find $E\left(y_i | x_i, c_1 < y_i < c_2\right)$ and $E\left(y_i | x_i\right)$. Use the fact that if $Z \sim N(0,1)$ then

$$E\left(Z | c_1 < Z < c_2\right) = \frac{\phi\left(c_1\right) - \phi\left(c_2\right)}{\Phi\left(c_2\right) - \Phi\left(c_1\right)}$$

where ϕ and Φ are the pdf and cdf of a standard normal. Describe estimators of β_0 based on the these conditional expectations functions.
(c) Describe an estimator of β_0 that does not use any parametric assumptions on the form of the conditional distribution function of ε_i given x_i. Make sure to state all crucial assumptions underlying the estimator.

PART IV

Question 1: True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

(i) The process X_t that satisfies $X_t + 1.9X_{t-1} + .88X_{t-2} = \varepsilon_t + .2\varepsilon_{t-1} + .7\varepsilon_{t-2}$ (where ε_t is white noise) is causal and invertible.
(ii) If a time series is ergodic (not just "ergodic for the mean") then it is also α-mixing.
(iii) If one regresses a random walk $y_{1,t}$ on its own lagged observation $y_{1,t-1}$ and on an independent random walk $y_{2,t}$

$$y_{1,t} = \widehat{\alpha} y_{1,t-1} + \widehat{\beta} y_{2,t}$$

then the theory of spurious regression implies that the OLS estimator $\widehat{\beta}$ of β is inconsistent.
(iv) In general, in HAC estimation, the bandwidth S_T has to grow to infinity as the sample size T goes to infinity to guarantee that the variance of the HAC estimator converges to zero.

Question 2:

a) Precisely state Donsker's theorem and the continuous mapping theorem with all the assumptions needed. What is the Beveridge-Nelson decomposition?
c) For $t = 1, ..., T$, let $u_t := (u_{1t}, u_{2t})'$ where $u_t = \Psi(L)\varepsilon_t$ for ε_t an i.i.d. (2×1) vector with mean zero, variance PP', and finite fourth moments. Assume further that $\{s\Psi_s\}_{s=0}^{\infty}$ is absolutely summable and that $\Psi(1)P$ is nonsingular. For $i = 1, 2$, define $\xi_{it} := \sum_{s=1}^{t} u_{is}$. Derive the asymptotic distribution of $T^{-3/2}\sum_{t=1}^{T}(\xi_{1t}, \xi_{2t})'$.

Question 3:

Show that for a covariance stationary process Y_t, the linear projection $\widehat{E}(Y_{t+1}|Y_t)$ of Y_{t+1} on a constant and Y_t is given by

$$\widehat{E}(Y_{t+1}|Y_t) = (1 - \rho_1)\mu + \rho_1 Y_t,$$

where $\mu := E(Y_t)$, $\rho_1 := \gamma_1/\gamma_0$, and γ_k, as usually, denotes the covariance of Y_t at lag k. Show that for an AR(2) process with AR parameters ϕ_i $(i = 1, 2)$, the implied forecast is

$$\mu + [\phi_1/(1 - \phi_2)](Y_t - \mu).$$

Question 4:

Explain briefly how the Phillips and Perron test for a unit root works in the regression model

$$y_t = \alpha + \rho y_{t-1} + u_t,$$

where the error term u_t is possibly serially correlated and heteroskedastic.