Fall 2003 UCLA Department of Economics
Written Qualifying Examination in ECONOMETRICS

Instructions:
Answer **ALL questions** in Parts I, II, and III
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

1 Part I

Question 1:

Suppose that you have a model

$$y_i = x_i \cdot \beta + \varepsilon_i$$

with the restriction

$$E\left[z_i \varepsilon_i\right] = 0$$

for some z_i such that $\dim(z_i) = 1$. Suppose that we do not observe the entire component of the vector (y_i, x_i, z_i). Instead, we observe (y_i, z_i) for $i = 1, \ldots, n$, and (x_i, z_i) for $i = n+1, \ldots, n+m$, where n and m are both "large". Propose an estimator for β that is consistent as both n and m grow to infinity.
Hint 1: If there were no missing variable problem, you would have used IV because it is consistent.
Hint 2: You do not need to worry about asymptotic normality.

Question 2:

Suppose that you are given a model

$$y_{gi} = x_{gi} \cdot \beta + \varepsilon_{gi} \qquad g = 1, \ldots, G; i = 1, \ldots, N$$

where it is assumed that

$$E\left[x_{gi} \varepsilon_{gi}\right] = 0$$

We will assume that <u>N is fixed</u> and G grows to infinity. We will assume that the vector

$$(y_{g1}, x_{g1}, \ldots, y_{gN}, x_{gN}) \qquad g = 1, 2, \ldots$$

is i.i.d. We do NOT assume that (y_{gi}, x_{gi}) are independent of $(y_{gi'}, x_{gi'})$ for $i \neq i'$. In other words, observations belonging to the same group g may be correlated with each other.

(a) Show that the OLS estimator

$$\widehat{\beta} = \frac{\sum_{g=1}^{G} \sum_{i=1}^{N} x_{gi} y_{gi}}{\sum_{g=1}^{G} \sum_{i=1}^{N} x_{gi}^2}$$

is consistent for β

(b) What is the asymptotic distribution of $\sqrt{G}\left(\widehat{\beta} - \beta\right)$? How would you estimate the asymptotic variance?

2 Part II

Question 1:

Provide an example where Maximum Likelihood can produce inconsistent estimators in the presence of incidental parameters.

Question 2:

Derive the conditional maximum likelihood estimator of β in the normal linear static panel data model:

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it} \qquad i = 1, ..., n; \qquad t = 1, ..., T$$

where $\varepsilon_{it} \sim Niid\left(0, \sigma^2\right)$ and independent of x_{is} at all leads and lags. Show that it is consistent when n is large relative to T.

Question 3:

(a) Write down a panel data sample selection model with additive individual effects.

(b) Write the log-likelihood of a random effects specification of the model that assumes a normal distribution for the composite error term.

(c) Describe Heckman's two-step method for estimating the model of part (b).

(d) Describe a method for identifying a fixed effects specification of the model that does not require the parametrization of the distribution of the model's unobservables. Make sure to state the important assumptions that allow identification and to provide a consistent estimator for the model's parameters of interest.

(e) What are the advantages and disadvantages of the approaches in (b) and (d)?

3 Part III

Question 1:

Consider the non-linear cross-sectional model

$$y_i = g(x_i; \theta_0) + \varepsilon_i, \tag{1}$$

for $i = 1, ..., n$, where x_i is a vector of regressors and θ_0 is the corresponding vector of parameters, both are $K \times 1$ vectors, and $g(\cdot; \theta)$ is a known function. It is assumed that this is the true model, that is,

$$E\left[y_i | x_i\right] = g(x_i; \theta_0).$$

(a) Provide the non-linear least-squares estimator for θ_0, say $\widehat{\theta}_n^{LS}$. Show that $\widehat{\theta}_n^{LS}$ is consistent estimator for θ_0 and has an asymptotic normal distribution, and provide the covariance matrix for that asymptotic distribution. Make sure you explain all the key assumptions needed for establishing consistency and asymptotic normality.

(b) One suggested an alternative way for obtaining the estimator in (a), by solving

$$\min_{\theta} \left(\frac{1}{n} \sum_{i=1}^{n} \varphi(y_i, x_i, \theta) \right)' W_n \left(\frac{1}{n} \sum_{i=1}^{n} \varphi(y_i, x_i, \theta) \right),$$

where

$$\varphi(y_i, x_i, \theta) = (y_i - g(x_i; \theta_0))\, h(x_i, \theta),$$

for some known vector-valued function $h(\cdot) \in R^M$, with $M > K$. Denote the suggested estimator by $\widehat{\theta}_n^W$. Show that $\widehat{\theta}_n^W$ is consistent estimator for θ_0 and has an asymptotically normal and provide the covariance matrix for that asymptotic distribution. Make sure you explain all the key assumptions needed for establishing consistency and asymptotic normality.

(c) Which of the two estimator, the one in (a) or the one in (b), would you prefer? Justify your answer in detail.

(d) Consider the estimator suggested in (b), only with the moment function given by

$$\varphi(y_i, x_i, \theta) = (y_i - g(x_i; \theta_0))\, m(y_i, x_i, \theta).$$

Can one obtaine a meaningful estimator by minimizing the same objective function as in (b)? Justify your answer.

(e) Suppose now that the function $g(\cdot; \theta_0)$ in (1) is not the true conditional mean of y_i, conditional on x_i. How would your answer to (a) change, if at all? Justify your answer.

Question 2:

Consider the censored linear regression model

$$y_i^* = x_i'\beta_0 + \varepsilon_i,$$

where y_i^* is a latent variable that is observed only if it exceeds a certain threshold, that is,

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* > a, \\ 0 & \text{otherwise.} \end{cases}$$

Also, x_i is a $K \times 1$ vector of exogenous regressors, while β_0 is a $K \times 1$ vector of unknown parameters, and $\varepsilon_i | x_i \sim \mathrm{N}(0, \sigma_\varepsilon^2)$.

(a) Which of the model's parameters are identified? Justify your answer.

(b) Write the likelihood function, say $L(\theta)$, where θ is the vector containing all the model's parameters that are identified. Show that the Maximum Likelihood estimator (MLE), say $\widehat{\theta}_n$, satisfies

$$\sqrt{n}\left(\widehat{\theta}_n - \theta_0\right) \xrightarrow{D} N\left(0, I^{-1}(\theta_0)\right),$$

where θ_0 is the population's true parameter vector and $I(\theta_0)$ denotes the Fisher's information matrix, evaluated at θ_0.

(c) Provide the Wald and Likelihood Ratio (LR) test statistic for testing the hypothesis H_0: $\sum_{k=1}^{K} \theta_{0j} = \mu_0$ for some known constant μ_0.

(d) Propose a way to test the hypothesis in (c) using the bootstrap method. When answering this question give enough details so that a programmer would be able to write a program to carry out the proposed procedure.

(e) Suppose now that

$$y_i^* = g\left(x_i'\beta_0\right) + \varepsilon_i,$$

for some known scalar function $g(\cdot)$, and all the other assumptions remained unchanged. Describe in detail how to obtain a symmetric confidence interval for the model parameters using the bootstrap method.