UCLA Economics

Econometrics Field Exam **4hours**

Fall 2001

Please answer four of the five questions. Use a separate blue book for each question.

1. A distribution that is used for count data, e.g., the number of patents, is the Poisson distribution. Suppose that conditional on R&D expenditures (X) the number of patents (Y) has a Poisson distribution with parameter $\beta_0 + \beta_1 X$.

   (a) Describe how you would estimate $\beta = (\beta_0, \beta_1)$ by maximum likelihood given a random sample of size N.

   (b) What is the variance of the maximum likelihood estimator?

   (c) Show that the least squares estimator for the regression of Y on a constant and X is consistent for (β_0, β_1)

   (d) How does the variance of the least squares estimator compare to that of the maximum likelihood estimator?

   (e) How can you modify the least squares estimator to make it efficient?

   (f) Suppose there is an additional regressor that affects the number of patents, e.g., quality of research. Assuming this is independent of expenditures X, how can you test for its presence using an information matrix test? You may assume that this variable enters in the same way as expenditures.

   Note: the probability function for a Poisson random variable Z with parameter λ is

   $$f_Z(z; \lambda) = \frac{\lambda^z \exp(-\lambda)}{z!}.$$

2. Consider the following linear model

   $$E[Y|X] = X\beta.$$

   Suppose we consider estimating β using a generalized method of moments framework with two moment functions

   $$\psi_1(Y, X, \beta) = X \cdot (Y - X\beta),$$

   and

   $$\psi_2(Y, X, \beta) = X - 3,$$

   where Y, X and β are all scalars.

(a) Describe the optimal gmm estimator for β.

(b) Describe the empirical likelihood estimator for β.

(c) Compare the variance of the optimal gmm estimator using both moments with the variance of the gmm estimator using only the first moment.

(d) Do the same for the case where the second moment is $\psi_2(Y, X, \beta) = Y - 4$. How do you interpret the differences.

3. Suppose that the Generalized Classical Normal Regression model applies to n observations from

$$y_i = x_i\beta + \varepsilon_i \qquad i = 1, ..., n$$

where the variance of ε_i is $\exp(w_i\alpha)$ for some observable non-stochastic $q-$dimensional variable w_i.

(a) Derive the GLS and ML estimators of β assuming α is known. What is their relationship?

(b) Describe the feasible GLS and ML approaches when α is unknown. Make sure to provide a consistent estimator of α for FGLS and to justify it.

(c) Derive the asymptotic distributions of the feasible GLS and ML estimators. What is their relationship?

4. Consider the panel data censored regression model (Type 1 Tobit):

$$y_{it} = \max(0, x_{it}\beta + \alpha_i + \varepsilon_{it}) \qquad i = 1, ..., N; t = 1, 2$$

where ε_{it} are i.i.d. over time conditional on α_i and (x_{i1}, x_{i2}), and α_i are individual specific effects. Throughout assume that sampling across individuals is random.

(a) Describe the (random effects) ML estimator of β under the assumption that $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$, $\alpha_i \sim N(0, \sigma_\alpha^2)$, and ε_{it} and α_i are uncorrelated for all i and all t.

(b) Describe the Heckman two-step estimator of β under the same assumptions as in part (a).

(c) Describe an estimator that is consistent for large N and fixed T and which is does not make any distributional assumptions on either α_i or ε_{it}.

5. Suppose (y_i, x_{1i}, x_{2i}) is an i.i.d. sequence with $E[y_i|x_{1i}, x_{2i}] = x_{1i}'\beta_1$ (i.e. $\beta_2 = 0$).

(a) Let $(\hat{\beta}_1, \hat{\beta}_2)$ be the OLS coefficients from a regression of y on (x_1, x_2). What is the plim of $\hat{\beta}_2$?

(b) Let $\tilde{\beta}_1$ be the OLS coefficient from a regression of y on x_1. Is $\tilde{\beta}_1 = \hat{\beta}_1$? What is the plim of $\tilde{\beta}_1$?

(c) Instead of regressing y on x_1, suppose we estimate β_1 using 2SLS with instruments (x_1, x_2). Call the 2SLS estimate $\bar{\beta}_1$. Is $\bar{\beta}_1 = \hat{\beta}_1$? Is $\bar{\beta}_1 = \tilde{\beta}_1$? What is the plim of $\bar{\beta}_1$?

(d) Now suppose the population covariance of X_1 and X_2 is zero; $Cov(x_1, x_2) = 0$. Does that change your answer to the previous part? If so, how? Explain your answer.

(e) Let $(\gamma_1, \gamma_2) = \text{plim } (\hat{\beta}_1, \hat{\beta}_2)$ (from part (a)). Suppose

$$\begin{pmatrix} \hat{\beta}_1 - \gamma_1 \\ \hat{\beta}_2 - \gamma_2 \end{pmatrix} \xrightarrow{d} N(0, V)$$

and for an estimator $\hat{V}$, plim $\hat{V} = V$. Define

$$\hat{\hat{\beta}}_1 = \arg\min_{b_1} \begin{pmatrix} \hat{\beta}_1 - b_1 \\ \hat{\beta}_2 - 0 \end{pmatrix}' \hat{V}^{-1} \begin{pmatrix} \hat{\beta}_1 - b_1 \\ \hat{\beta}_2 - 0 \end{pmatrix}$$

Let $\delta = \text{plim } \hat{\hat{\beta}}_1$. What is δ? What is the limiting distribution of $\sqrt{n}(\hat{\hat{\beta}}_1 - \delta)$?