QUALIFYING EXAM IN MACROECONOMICS

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Instructions: You have four hours to complete this exam. There are three parts. Each has equal weight. **Answer each part in a separate blue book.**

Part One (*Blue Book Number 1*)

Consider an economy consisting of a large number of identical farmers that are each endowed with 1 units of land at the beginning of time. The farmers use their own land and labor to produce a consumption good in order to maximize preferences given by $E\sum_{t=0}^{\infty}\beta^t\left(\log c_t - Ah_t\right)$, where A is a positive constant, $0<\beta<1$, c_t is consumption in period t, and h_t is hours worked. A farmer's production function is given by $y_t = z_t L_t^{\theta} h_t^{1-\theta}$, where $0<\theta<1$. Here, L_t is the quantity of land owned by the farmer in period t and z_t is an i.i.d. random technology shock. The shock z_t, which is revealed at the beginning of period t, can take two values, $\mu+\sigma$ or $\mu-\sigma$. The probability of each is .5. All output must be consumed (by someone) in period t or disposed of; it cannot be stored. In addition, land cannot be produced and does not depreciate.

A. Formulate the problem that would be solved a social planner in this economy. Solve for the optimal decision rules for consumption and hours worked.

B. Consider now a version of this economy in which there are markets for goods, a market for land, and a market for state contingent claims. Define a *recursive competitive equilibrium* for such an economy. Include only the markets specified; do not add additional markets. For notational uniformity, let $p(z)$ be the price of land and $q(z',z)$ the price of contingent claims.

C. Solve for the equilibrium defined in part B. In particular, find the equilibrium pricing functions for land and contingent claims. Are the equilibrium quantities (for consumption and hours worked) the same as that chosen by the social planner? Explain.

D. Compute the *average* land price, $\overline{p}$. Consider an option that entitles the owner to exercise the right to buy, but only if the owner chooses to do so, one unit of land one period into the future at the price $\overline{p}$. Compute the price of this option. What is the expected rate of return on an option purchase given the current state z? What is the average rate of return across all states?

E. Compare the average return on the option investment computed in part (D) with the average rate of return on a risk-free bond. Which is larger?

Part Two (*Blue Book Number 2*)

Consider the following model of endogenous growth in discrete time.

There are two types of goods: a homogenous commodity Y which can either be consumed or accumulated, to be used in production next period; human capital H which can be accumulated, to be used in production. All markets are perfectly competitive.

Homogenous consumers live three periods. During the first period they are endowed with an exogenous level h^y of human capital; using this and real resources $d > 0$ they can accumulate human capital for next period. During the second period they sell their human capital, save and consume. During the third they consume out of their saving.

In each period $t = 0; 1; 2; :::$, physical capital, k_t, and human capital, h_t, are owned, respectively, by the old and the middle age individuals. Aggregate output of the homogenous commodity is $y_t = F(h_t; k_t)$, where $F(h; k)$ is a constant return to scale and neoclassical production function.

The accumulation of human capital by the young is represented by the function $h(d; h^y)$, which is a constant return to scale neoclassical production function. Assume that $h_t^y = \delta h_t$, $\delta > 0$ that is the endowment of human capital of the new generation is a constant fraction of the human capital accumulated by the previous one.

We assume agents draw utility from $(c_t^m; c_{t+1}^o)$ (consumption when middle age and old), according to $u(c_t^m) + \beta u(c_{t+1}^o)$. u( ) is a standard utility function.

Let the homogenous commodity be the numeraire. In each period $t = 0; 1; 2; :::$, output y_t is allocated to three purposes: aggregate consumption $(c_t = c_t^m + c_t^o)$, accumulation of next period's physical capital, (k_{t+1}) and investment in human capital (d_t). Human and physical capital are hired by firms at competitive prices equal, respectively, to $w_t = F_1(h_t; k_t)$ and $1 + r_t = F_2(h_t; k_t)$. Aggregate saving is allocated, through competitive credit markets, to finance investments in physical and human capital $(s_t = k_{t+1} + d_t)$, accruing a total return equal to $(1 + r_{t+1})s_t = R_{t+1}s_t$.

(1) Write down the life-cycle optimization problem for the agent born in period t.
(2) Define an intertemporal equilibrium for this model.
(3) Derive the first order and market clearing conditions determining the competitive equilibrium over time. See how far you can characterize the competitive equilibrium without assuming any functional form for utility and production functions.
(4) Assume the utility function is logarithmic and the two production functions are Cobb-Douglas, with different parameters. Check if a balanced growth path exists. What are the properties of the balanced growth path?
(5) Characterize the equilibrium dynamics in the case of (4). Do you always converge to the balanced growth path?

(6) What happens in this economy if the credit markets for lending to young agents are shut down? How would you try "replacing" them? Be precise and brief on this last issue.

Part Three (*Blue Book Number 3*)

Consider a productive OLG economy with constant population (one individual per generation) where agents live for two periods, they are endowed with one unit of labor (offered inelastically) in young age only, they consume in old age only and they have the following utility function:

$$U(c_t, c_{t+1}, m_t) = c_{t+1} + L(m_{t+1}), \qquad \beta > 0,$$

where c is consumption, m is real money balances and $L(.)$ defined in R_+ satisfies the following assumptions for all $m \geq 0$:

$$L'(m) > 0, \qquad L''(m) < 0, \qquad \lim_{m \to 0} L'(m) = \infty.$$

Capital fully depreciates in one period and the production function $F(k, L) = Lf(k/L)$ satisfies constant returns to scale and strict concavity in capital k and labor L.

The nominal stock of money issued by the government in any period t is M_{t+1}, where $M_1 > 0$ and $M_{t+1} = \mu M_t$, $\mu \geq 1$. When $\mu > 1$, the government makes a money transfer $H_t = M_{t+1} - M_t$ to the old alive in period t.

Now denote the gross real return on capital at time t as R_t and let $p_t/p_{t-1} = \pi_t$. Then, the budget constraints of a consumer born at time t are:

$$m_{t+1}\pi_{t+1} + k_{t+1} = w_t, \qquad c_{t+1} = R_{t+1}k_{t+1} + m_{t+1} + h_{t+1}$$

where w is the real wage and h is the money transfer in units of the consumption good.

1. Determine the competitive equilibrium of this economy for $\mu = 1$, in terms of the variables k and m, i.e., write down the system of equations defining the equilibrium values of (k_{t+1}, m_{t+1}) for given (k_t, m_t).

2. Assume that the production function is Cobb-Douglas and derive the approximate planar phase diagram in the space (k, m), i.e., study the locus of points in (k, m) where $k_{t+1} = k_t$ and $m_{t+1} = m_t$. What can you say about the position of the steady states of the system? Assuming that the system has only one steady state with positive real money balances (monetary steady state), what can you say about the stability of the monetary steady state?

3. Now assume $\mu > 1$ and $h > 0$ and write down the equilibrium dynamics in k and m corresponding to this situation. Then, reproduce the approximate phase diagram in (k, m) in this case, characterize the steady states and identify the seignorage of the government.

4. Discuss the possible effects of an increase in μ on the steady state capital stock and real money balances assuming that the monetary steady state is unique. Under what conditions the inflation rate may be expansionary? (Hint: use the fact that the uniqueness of the monetary steady state implies some conditions on the derivatives of $L(m)$ and $f(k)$). Check your answer in the case in which $L(m) = \gamma \ln m$.

5. Evaluate the effect of an increase in μ on the agent's utility in a steady state, i.e., on:

$$U = c + L(m) = Rk + m + h + L(m).$$

Prove the following proposition: when inflation is expansionary, a necessary condition for μ to have a positive effect on U is:

$$f'(k) > 1.$$

Give an intuitive argument to show that the Friedman Rule may not apply in this model.