

**Macroeconomic Theory  
Ph.D. Qualifying Examination**

**Comprehensive Examination**

**UCLA Dept. of Economics**

- You have 4 hours to complete the exam.
- There are three parts to the exam.
- Answer all parts. Each part has equal weight.

**ANSWER EACH PART IN A SEPARATE BLUE BOOK.**

**PART ONE: ANSWER IN BOOK 1  
WEIGHT 1/3**

1) This question is about the  $Ak$  model and the role of gestation lags in economic growth. For all parts of it, you will assume that the aggregate production function is  $Y_t = AK_t$  for  $A > 0$ .

- A. Show that all dynamical equilibria in the Solow growth model with this technology are balanced growth paths. Compute the growth rate.
- B. Does the  $Ak$  technology generate balanced growth equilibria in the optimum growth model? If so, compute the appropriate growth rate.
- C. Assume that:

$$s_t = sY_t$$

where  $0 < s < 1$ . Suppose that current saving,  $s_t$ , becomes productive capital with a lag of two periods rather than with a normal lag of one period. In other words, let

$$K_{t+1} = (1 - \delta)K_t + s_{t-1}$$

Examine once more the dynamics of the descriptive growth model. Find its asymptotic growth rate. How does the economy converge to this rate?

- D. Repeat part C for an optimum growth model with an isoelastic flow utility function of the general form  $u(c) = c^{1-\gamma} / (1-\gamma)$ ,  $\gamma \geq 0$ .
- E. Explain intuitively why your answers to parts C and D differ.

**PART TWO: ANSWER IN BOOK 2**  
**WEIGHT 1/3**

2) The economy to be studied in this question is a one-sector stochastic growth model with vintage capital. That is, instead of capital depreciating linearly over time, we will assume that capital remains productive for a finite number of periods and then turns to dust.

The preferences of an infinitely lived representative agent are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - h_t),$$

where  $0 < \beta < 1$  and  $U$  is concave, increasing and continuously differentiable in both arguments. We will assume initially that capital is productive for two periods. Let  $k_1$  be the stock of capital in its first period of use and  $k_2$  be the stock of capital in its second period of use. Output is produced using the technology,  $z_t F(k_{1t}, k_{2t}, h_t)$ , where  $z_t$  is a random variable that follows a Markov process with a conditional distribution function,  $G(z_{t+1}; z_t)$ . Output can be allocated to current consumption or to new capital that is available for production the following period. Finally, we assume that the capital stock depreciates at the rate  $(1 - \mu)$ . In particular,  $k_{2,t+1} = \mu k_{1t}$ ,  $\mu < 1$ .

- A. Formulate the dynamic program that would be solved by a social planner in this economy. Be complete.
- B. Repeat part (A) for an economy in which capital can be used for  $n$  periods.
- C. Derive the first order necessary conditions and envelope conditions that characterize a solution to the problem of part (A).
- D. Provide a set of equations the solution to which is the steady state values for  $c, h, k_1$ , and  $k_2$ . You should assume that the unconditional mean of  $z$  is 1.
- E. Define a *Recursive Competitive Equilibrium* for the economy of part (A). In your decentralization, assume that there is a market for final output, rental market(s) for capital, a labor market, and a resale market for existing capital held after production takes place.
- F. Derive an equation that characterizes the equilibrium price of existing capital in terms of current state variables (of course, you cannot solve for this explicitly since you are given only general functional forms). What is this price in steady state?
- G. Suppose now that the two vintages of capital are perfect substitutes in production. That is, suppose that the production function is  $z_t F(k_t, h_t)$ , where  $k_t = k_{1t} + k_{2t}$ . Evaluate the following reasoning: "If existing capital ( $k_2$ ) is a perfect substitute for new capital in production, then existing capital must sell for the same price as new capital. Hence, the price of exiting capital must be equal to 1."

**PART THREE: ANSWER IN BOOK 3**  
**WEIGHT 1/3**

This part of the exam consists of 5 statements. State whether each of them is true false or uncertain. In each case state your reasons – no marks will be given for unsubstantiated answers.

A. The reduced form of a rational expectations macroeconomic model is represented as:

$$\begin{bmatrix} k_t \\ c_t \end{bmatrix} = A E_t \begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} + B \begin{bmatrix} u_{t+1} \\ v_{t+1} \end{bmatrix}$$

where  $k_t$  and  $c_t$  are the log deviations of capital and consumption from a steady state,  $u_{t+1}$  is an i.i.d. fundamental shock to technology with zero mean and  $v_{t+1}$  is a non-fundamental error. The matrices  $A$  and  $B$  are square and of full rank and  $A$  is given by:

$$A = \begin{bmatrix} 1.2 & 1 \\ 0.5 & 0.5 \end{bmatrix}.$$

**This model has a locally unique rational expectations equilibrium. True false or uncertain?**

B. Consider an overlapping generations model in which there is a single perishable good, and a single type of agent who lives for two periods. Population is growing at rate  $g$  so that the number of young people is always equal to  $(1+g)$  times the number of old. There is no uncertainty and each agent has a utility function described by:

$$U = c_t^t - \frac{(2 - c_{t+1}^t)^2}{2}.$$

where the superscripts index date of birth and subscripts index date of consumption. The initial old has an endowment of 1 unit of the perishable good. All other agents have an endowment  $\{a, 1\}$  where  $a$  is a positive number.

**In this model there is a steady state equilibrium in which money has positive value if and only if  $g$  is bigger than zero. This conclusion is independent of the value of  $a$ . True false or uncertain?**

C. Consider a pure trade general equilibrium model with two periods and  $n$  agents. In period 1 there are 4 states of nature. In period 1 there is trade in financial assets but no trade in commodities. In period 2 there is a single commodity in each state. There are two types of agents. Type 1 is born in period 1 and lives for two periods. Type 2 is born in period 2. There is no fundamental uncertainty. Agents may trade a complete set of Arrow securities in period 1.

**In this model sunspots cannot matter in the sense of Cass and Shell. True false or uncertain?**

- D. Consider a representative agent growth model in which there are many identical infinitely lived agents. Each agent produces output using the technology

$$Y_t = A(K_t^\rho + L_t^\rho)^{\frac{1}{\rho}}, \quad \rho < 1.$$

where  $A$  is a positive constant,  $Y_t$  is output,  $K_t$  is capital and  $L_t$  is labor. The utility function of the representative agent is given by

$$U = \sum_{t=1}^{\infty} \beta^t \log C_t,$$

where  $C_t$  is consumption. Output may be consumed or invested in capital that depreciates at rate  $\delta$ . Each agent is endowed with a single unit of labor in each period and with a single unit of capital in the first period.

**In this model there will be sustained growth if  $1+A-\delta$  is bigger than  $1/\beta$ . This conclusion is independent of the other parameters of the model. True false or uncertain?**

- E. Consider a model of economic growth in which a single final good is produced from a large number of intermediate goods using the technology:

$$Y = \left( \sum_{i=1}^n Y_i^{1/2} \right)^2.$$

where  $\rho$  is a number between zero and one and  $Y_i$  is use of the  $i$ 'th intermediate good. Each intermediate good is produced by a monopolistic competitor using the technology:

$$Y_i = K_i - 1$$

where  $K_i$  is capital used by the  $i$ 'th intermediate producer. The economy as a whole has 4 units of capital and factor markets are competitive.

**In this economy there will be 2 intermediate producers in equilibrium. True false or uncertain?**