UCLA Department of Economics **Spring 2013**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider an economy with a representative household with N_t identical members. The household's preferences are given by,

$$\sum_{t=0}^{\infty} \beta^t N_t \left\{ \log c_t + A \log(1 - h_t) \right\}.$$

Each member of the household is endowed with 1 unit of labor each period. The number of members evolves over time according to the law of motion, $N_{t+1} = \eta N_t$, $\eta > 1$.

Output is produced using the following technology:

$$Y_t = \gamma^t e^{z_t} K_t^{\mu} (N_t h_t)^{\phi} L_t^{1-\mu-\phi}$$

Here, $\gamma > 1$ is the gross rate of exogenous total factor productivity growth, K_t is total (<u>not</u> *per capita*) capital, Y_t is total output, and L_t is the total stock of land. Land is assumed to be a fixed factor; it cannot be produced and does not depreciate. To simplify with out loss of generality, assume that $L_t = 1$ for all *t*.

The variable z_t is technology shock that can take on two possible values and evolves according to a symmetric Markov chain. The unconditional mean of z_t is zero, its unconditional variance is σ^2, and $E(z_t z_{t+1}) = \rho$.

The resource constraint, assuming 100 percent depreciation of capital each period, is given by

$$N_t c_t + K_{t+1} \leq Y_t \ .$$

A. Formulate the social planning problem for this economy as a *stationary* dynamic program. Be clear about the transformation performed so that all variables are stationary.

B. Characterize the balanced growth path of this economy. That is, find expressions that determine c_t h_t and K_t along this growth path. In particular, solve explicitly for the growth rate of this set of variables.

C. Suppose that a period is one quarter and suppose you are given annual growth rates for population and per capita income. You are also given values for factor shares, the average amount of time spent working and the annual capital-output ratio. Show how these facts can be used to calibrate γ, η, A and β.

D. Calibrate the state space and transition matrix *P* of the Markov for *z* chain given values for σ and ρ.

E. Define a *recursive competitive equilibrium* for this economy assuming markets for labor, consumption goods, land, and capital services.

1 Part 2. The Transition from Farming to Manufacturing in a Growing Economy

There are 2 consumption goods: an agricultural good (f_t)and a manufactured good (m_t). Preferences for the representative household are given by:

$$\max \sum \beta^t \{\ln(f_t) + m_t + \ln(1 - l_a - l_m)\}$$

The technologies are

$$M_t = A_t L_{Mt}$$

$$F_t = A_t L_{At}$$

Technological progress evolves as:

$$A_t = A_0(1+\gamma)^t, \gamma > 0, t = 1, 2, ...$$

(1) Write this as a social planning problem amd solve for the planner's first order conditions (10 points)

(2) Show that that there exists a level A_t^* such that for $A_t \leq A_t^*$,the economy will produce only agricultural goods. How much labor is employed in this case? Show that labor supply is constant in the interval $(0, A_t^*]$.(10 points)

(3) Show that for $A_t > A_t^*$, both manufactured and farm goods are produced, and that the fraction of employment devoted to farm goods approaches zero asymptoticaly. What happens to total labor supply asymptoticaly? What features of the model economy drive these results? (10 points)

(4) Decentralize this problem as a competitive equilibrium and show formulas for equilibrium prices, choosing the numeraire as the price of the agricultural good. (10 points)

(5) Why doesnt the relative price of the manufactured good rise as the economy becomes wealthhy and demand for this good rises? (5 points)

Part III

Shocks to the relationship between Vacancies and Unemployment: One of the puzzling features of the labor market in this recession is that the ratio of vacancies to unemployed workers is high relative to historical experience and yet employment is not recovering quickly. We explore how this might happen in a Mortensen-Pissarides search model.

Time is continuous and labelled $t \geq 0$. There is a measure one of workers. At each date t, the measure of workers that are unemployed is denoted $u_t \in [0,1]$ while the measure of workers who are employed is $1 - u_t$. To hire workers, firms post vacancies. Let v_t denote the measure of vacancies posted at date t. The measure of matches between workers and vacancies at t is given by $m(u_t, v_t)$ with the standard properties (non-negative, increasing and strictly concave in each argument, $m(0, v) = m(u, 0) = 0$, and $m(u.v)$ constant returns to scale). We assume that each unemployed worker finds a match at t at rate $\alpha_{ut} = m(u_t, v_t)/u_t$ and each firm with a vacancy finds a match at t at rate $\alpha_{et} = m(u_t, v_t)/v_t$. From here forward, we will consider steady-states of this economy in which u_t, v_t and the corresponding rates α_{ut} and α_{et} are constant over time.

We assume that unemployed workers consume b per unit time while unemployed and that firms that have posted a vacancy must pay $k > 0$ per unit time to post that vacancy. The productivity of a match between any given worker and a firm has productivity $y > b$ with probability γ and productivity $0 < b$ with probability $1 - \gamma$. Once the match is formed, this productivity is permanent until the match dies. That is, a match with productivity $y > b$ has this productivity until it switches to become a match with productivity 0 at rate $\lambda > 0$ per unit time. Matches with productivity 0 always have productivity zero.

When an unemployed worker and a firm with a vacancy are matched, they first learn the productivity of their match and then bargain over the wage w that the firm will pay the worker. If the match has productivity y, the profits to the flow profits to the firm are $y - w$. In this case, the worker and the firm bargain over the wage, and we assume that the wage they agree on is equal to the Generalized Nash Bargaining solution to their match with bargaining weight $\theta \in (0, 1)$ for the worker and $1 - \theta$ for the firm. If the match has productivity 0, there is no surplus to be divided between the worker and the firm, so the worker continues as unemployed and the firm continues to look for a new match with another worker.

Part A: Let $W(w)$ denote the value to a worker of being employed at wage w and let U denote the value to the worker of being unemployed. Let $J(y - w)$ denote the value to a firm of having a worker with productivity y hired at wage w and V denote the value of an unfilled vacancy. Assume that the worker and the firm both discount the future at rate $r > 0$. Write the Bellman Equations for the firm and the worker defining $W(w)$, $J(y - w)$, U, and V. Take care to consider the fact that the quality of the match is random. Also derive a formula to describe the surplus of a match with productivity y given by

$$S = W(w) - U + J(y - w) - V$$

Part B: Use the fact that the wage must satisfy

$$w \in argmax_w \left(W(w) - U\right)^{\theta} \left(J(y - w) - V\right)^{(1-\theta)}$$

to show that

$$W(w) - U = \theta S$$

and

$$J(y - w) - V = (1 - \theta)S$$

Part C: Under the assumption that there is free-entry into creating vacancies, so the value of a vacancy must be equal to zero in any period in which there are positive vacancies, show how to characterize the equilibrium vacancy-unemployment ratio v/u.

Part D: Show how to characterize the corresponding job finding rate α_u, and the corresponding steady-state unemployment rate u.

Part E: What happens to the equilibrium vacancy unemployment ratio and the steady-state level of unemployment when b increases?

Part F: What happens to the equilibrium vacancy unemployment ratio and the steady-state level of unemployment when y increases?

Part G: What happens to the equilibrium vacancy unemployment ratio and the steady-state level of unemployment when γ increases?

Part H: Is it possible to have the equilibrium vacancy unemployment ratio increase and the steady-state level of unemployment increase when y increases and γ decreases? What conditions would need to be satisfied for this to happen?