UCLA Department of Economics   **Spring 2012**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider a real business cycles model with a representative household that lives forever and maximizes the following utility function:

$$E\sum_{t=0}^{\infty} \beta^t \{\log c_t + A\log L_t\}, \quad 0 < \beta < 1 \text{ and } A > 0.$$

Here, c_t is consumption and L_t is a convex combination of leisure in periods t and $t-1$. Each period, households are assumed to have one unit of time that can be allocated between market work, h_t, and leisure. In particular, let $L_t = a(1-h_t) + (1-a)(1-h_{t-1})$ where $0 \le a \le 1$.

Output, which can either be consumed or invested, is produced according to a constant returns to scale technology, $y_t = e^{z_t}k_t^{\theta}h_t^{1-\theta}$, where y_t is output and k_t is the stock of capital. The variable z_t is a technology shock observed at the beginning of period t that evolves through time according to a first order autoregressive process with mean zero innovations. The stock of capital is assumed to depreciate at the rate δ each period.

(a) Write the social planners problem for this economy as a dynamic programming problem.

(b) Derive as set of equations that characterize a sequence $\{c_t, h_t, L_t, k_{t+1}, y_t\}_{t=0}^{\infty}$ that solves this problem. Be sure that you have the same number of equations as unknowns. Explain the role of the *transversality condition* in determining this optimal sequence.

(c) Define a *recursive competitive equilibrium* for this economy.

(d) Are equilibrium allocations the same as the ones that solve the social planning problem? If they are the same, provide an example of how the economy could be modified so that equilibrium and planner allocation do not coincide.

(e) Derive a log-linearized version of the first order condition for hours worked for the problem described in part (a). Using this and other log-linearized equations as needed, derive an expression for $\tilde{h}_t \equiv \log h_t - \log \overline{h}$ as a linear function of $\tilde{k}_t, \tilde{c}_t, z_t,$ and $\tilde{h}_{t-1}$.

(f) In a standard real business cycle model, $a = 1$. Using the equation derived in part (e), explain how setting $a < 1$ might change the cyclical properties of the model economy.

(g) Describe how you would calibrate the parameter a? What data would you use? Are steady state growth facts useful for this purpose? Explain.

Part II

Money and Neoclassical Growth

A representative household maximizes the following lifetime utility function

$$\max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) - \phi \frac{h_t^{1+\sigma}}{1+\sigma}\}, 0 \leq \sigma < \infty$$

Consumption is subject to a cash-in-advance constraint, in which accumulated cash (m_t) plus a transfer from the government $(\bar{m}_t)$ is used to purchase consumption.

$$c_t \leq \frac{m_t}{p_t} + \frac{\bar{m}_t}{p_t}$$

Aggregate production is given by:

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta} = C_t + I_t,$$

where upper case letters denote per-capita values of the variables. Each household has one unit of time per period.

The aggregate capital stock evolves in the standard way:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

The aggregate stock of money evolves over time according to the following government rule:

$$M_{t+1} = M_t(1 + \mu)$$

The consumer's wealth constraint is given by:

$$\frac{(1+i_t)b_t}{p_t} + w_t h_t + (r_t + 1 - \delta)k_t + \frac{m_t}{p_t} + \frac{\bar{m}_t}{p_t} \geq c_t + k_{t+1} + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t}, \ k_0, m_0 \ given$$

(Note that the object b_t represents borrowing/saving between consumers. Since everyone is identical, there will be no borrowing/.saving in this asset, but there will be an equilibrium nominal interest rate for this asset that insures that all households are indifferent between borrowing or saving in this asset).

(1) Define a competitive equilibrium for this economy, and show that money is neutral.

(2) Solve for the value of μ such that the steady state allocations in this monetary economy are the same as those from the nonmonetary version of this

economy, which is the economy with the same preferences, production technology, and capital accumulation technology, but without the CIA constraint. Denote this value of μ as μ^*.

(3) Solve for the steady state rate of change in the price level, $\frac{p_{t+1}}{p_t}$, and the steady state nominal interest rate at each point in time, i_t. In what sense can we say that this monetary policy is optimal?

(4) Consider a steady state with $\mu > \mu^*$. Show that the CIA constraint binds. Show that steady state output, consumption, labor, and capital are all lower than the allocations in the μ^* economy.

(5) Show that the nonmonetary version of this economy in steady state, with a constant labor income tax, is observationally equivalent to the monetary economy with $\mu > \mu^*$. What is the intuition behind this result? Show how the parameter σ governs the distortion imposed by money (and the labor tax) in these economies.

(6) Suppose that the government picked a constant rate of money growth, $\mu < \mu^*$. Does a steady state equilibrium exist?

(7) Consider fixing the value of $\mu > \mu^*$, and fix the value of β, and assume that the time period is one year. Now, consider shortening the time period to one week, but keeping the same *annual* money growth rate and the same *annual* household discount rate. What happens to the distortion imposed by money as the time period becomes shorter?

Part III

Fiscal Policy in an Overlapping Generations Model:

Consider the following overlapping generations model. Time is discrete and denoted $t = 1, 2, 3, \ldots$. In period 1, there is an initial cohort of old agents of measure 1. These agents are endowed with the initial capital stock k_0 and the initial stock of government debt b_0. These agents have utility which is strictly increasing in consumption c_1^o. Every period $t \geq 1$, a cohort of measure 1 of two-period lived agents is born. These agents are each endowed with 1 unit of labor when young, which they supply inelastically. These agents have utility over their consumption while young, c_t^y, and their consumption while old c_{t+1}^o, given by

$$\log(c_t^y) + \beta \log(c_{t+1}^o)$$

Each period, there are firms that rent physical capital and hire labor to produce output according to

$$y_t = A k_{t-1}^\alpha l_t^{(1-\alpha)}$$

The firm pays rental payments $r_t k_{t-1}$ to the owners of the capital stock at t (the old at t) and wage payments $w_t l_t$ to the young at t. The firm chooses k_{t-1} and l_t to maximize profits given rental rate r_t and wage rate w_t each period.

The government consumes spending every period g_t. The government finances this expenditure with taxes on labor and with debt. The government collects $\tau_t w_t l_t$ in tax revenue from young workers' wages and these workers take home $(1 - \tau_t) w_t l_t$ in after tax wages. The government sells debt b_t at price q_t. In period t, the debt pays off b_t in goods in period $t+1$. The government budget constraint is given by

$$q_t b_t + \tau_t w_t l_t = g_t + b_{t-1}$$

for al $t \geq 1$.

Part A: Define an allocation. Define a feasible allocation.

Part B: Define a competitive equilibrium (without money) in this environment.

Part C: Use the definition of a competitive equilibrium to derive a pair of first order difference equations describing the evolution of the physical capital stock k_t and government debt b_t over time. You will get partial credit if you describe the first order difference equation that the stock of physical capital must satisfy under the assumptions that initial debt $b_0 = 0$ and that the government chooses taxes τ_t each period to balance the budget so that $b_t = 0$ for all t.

Part D: Use these difference equations to describe the impact on current output y_t in period t of an increase in taxes τ_t, an increase in government expenditure g_t, and an increase in the stock of government debt carried into the period b_{t-1}. Use these difference equations to describe the impact on next period's capital stock k_t

of an increase in labor taxes τ_t, an increase in government expenditure g_t, and an increase in the stock of government debt carried into the period b_{t-1}.

Part E: Is it necessarily the case that the discounted present value of government expenditure is equal to the discounted present value of taxes in a competitive equilibrium in this economy? If you believe that it is possible for these two not to be equal, please explain. If these two are not equal, then what limits government spending and taxes in the long run? Can the government spend whatever it wants?