Question 1

Consider the following economy. There is a continuum of measure one of fishermen living at the same lake with preferences defined by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \gamma \log(1 - n_t)\right),$$

where $c_t \geq 0$ denotes the individual consumption of fish, $0 < n_t < 1$ is labor supply, and the parameters satisfy $0 < \beta < 1$ and $\gamma > 0$. The total amount of fish left in the lake at time t is denoted F_t, and C_t is the aggregate catch, i.e., the total amount of fish removed from the lake at time t. Fish neither reproduce nor die of natural causes, so that the law of motion for fish is:

$$F_{t+1} = F_t - C_t.$$

The only use of fish is for consumption, and since there is measure one of people we have $c_t = C_t$. The size of catch depends on the fishermen's labor supply. Let N_t denote the aggregate labor supply. The aggregate catch is given by:

$$C_t = N_t F_t.$$

That is, if all fishermen work all the time, all fish are caught. The aggregate production is linear in labor. Accordingly, from the perspective of a single fisherman individual output is given by:

$$c_t = n_t F_t.$$

Part A: Provide a Bellman equation for the social planning problem faced by a planner who can set n_t (and thus $N_t = n_t$) for all fishermen simultaneously. Show (by using guess-and-verify) that the value function is log-linear in fish. Find the optimal labor supply n_t, and solve for the value function.

Part B: Now assume that there is no central planner. There are also no property rights over fish; instead, the stock of fish is a collectively used resource. All fishermen decide on their labor supply n_t individually, without taking into account the effect that their fishing imposes on others. Define an equilibrium, and provide a Bellman equation for the choice problem of an individual fisherman. What is the equilibrium labor supply n_t, and how does it compare to the planning solution? Explain the intuition for your result.

Part C: Can you think of a taxation scheme that implements the socially optimal allocation as an equilibrium?

Question 2

Find the equilibrium solution to the log-linearized first order conditions of the following stochastic growth model. Assume that consumers have preferences of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t)$$

with

$$u(c, 1 - l) = \log(c) + \frac{\gamma}{1 - \nu}(1 - l)^{1-\nu}$$

The resource constraint is given by

$$c_t + k_t - (1 - \delta)k_{t-1} = A_t k_{t-1}^{\alpha} l_t^{(1-\alpha)}$$

with A_t representing the current value of Total Factor Productivity at t. Assume that the logarithm of A_t evolved according to

$$\log(A_{t+1}) = (t + 1)\mu + \rho(\log(A_t) - t\mu) + \epsilon_{t+1}$$

with ϵ_{t+1} drawn independently each period from a Normal Distribution with mean zero and variance σ^2.

Assume that $\beta, \rho, \alpha \in (0, 1)$ and $\nu > 0$.

Part A: Over the long run, what is the average growth rate of output in this economy in terms of the parameters above? Choose a transformation of the variables in this economy that will render the variables stationary and write down the equations of the model that need to be log-linearized in terms of these variables.

Part B: Describe in detail how to find a solution to the model of the form

$$\hat{k}_t = P\hat{k}_{t-1} + Q\hat{A}_t$$

$$\hat{c}_t = R_c\hat{k}_{t-1} + S_c\hat{A}_t$$

$$\hat{l}_t = R_l\hat{k}_{t-1} + S_l\hat{A}_t$$

In particular, show how to solve for P, Q, R_c, R_l, S_c, S_l in terms of the parameters above and the steady-state fractions of output consumed $\frac{\bar{c}}{\bar{Y}}$, the capital-output ratio $\frac{\bar{k}}{\bar{Y}}$ and time worked $\bar{l}$.

Question 3

In this problem, we examine how altruism of parents towards children affects the aggregate capital stock.

Consider an overlapping generations model in which time is denoted $t = 1, 2, 3, \ldots$. Every period t, a new generation is born and lives in periods t and $t+1$. At date $t = 1$ there is an initial generation of old agents who live only at that date. The size of the population of each generation of agents is constant and fixed at 1.

At each date t, the consumption of agents born at t (the young) is denoted c_t^y while the consumption of agents born in the previous period (the old or the initial generation at $t = 1$) is denoted c_t^o. Agents born at t have preferences over consumption at t and $t+1$ given by

$$\log(c_t^y) + \beta \log(c_{t+1}^o)$$

while the initial generation has preferences given by

$$\log(c_1^o)$$

Each agent born at t has one unit of labor that they supply inelastically at t. Thus, the aggregate supply of labor $l_t = 1$ every period.

Output in this economy at t is produced using physical capital invested in the previous period k_{t-1} and labor l_t according to the production function

$$y_t = k_{t-1}^{\alpha} l_t (1-\alpha)$$

We assume that this output is produced by competitive firms that rent capital at rental rate r_t and pay wages w_t.

The initial generation alive only at $t = 1$ is endowed with the initial capital stock k_0

The aggregate resource constraint is given by

$$c_t^y + c_t^o + k_t - (1-\delta)k_{t-1} = y_t$$

Part A: Define a competitive equilibrium in this environment, characterize the equilibrium and solve for the steady-state capital stock as a function of the parameters β, α and δ.

Part B: Now assume that the old in each period t are altruistic in that at the end of period t, they give fraction ν of their holdings of undepreciated physical capital to the young. The young derive no income from this inhereted capital at t, but they do get to rent it out at $t+1$. Define a competitive equilibrium in this environment with altruism, characterize the equilibrium and solve for the steady-state capital stock as a function of the parameters β, α, δ and ν. Does the steady-state capital stock rise or fall relative to the economy without altruism? Explain your answer.

Question 4

In this problem, we consider the impact of a tax on value added on aggregate output and productivity in a model with heterogeneous firms.

Consider a model in which time is discrete and denoted by $t = 0, 1, 2, 3, \ldots$. There is a representative household with preferences over sequences of consumption $\{c_t\}$ given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

This household has one unit of labor that it supplies inelastically to the market every period.

Output is produced in a continuum of heterogeneous competitive firms. Each firm is indexed by its current level of productivity z. A firm with productivity z can produce output with labor according to

$$y = z^{(1-\nu)} l^{\nu}$$

At the end of each period t, each firm either dies with probability δ or continues next period with the same productivity z with probability $(1-\delta)$.

At date $t = 0$, there number of firms is given by N_0 and the density of productivities across firms is given by $f_0(z)$.

New firms can be created with an expenditure of n_e units of labor per new firm. Specifically, to create M_t new firms starting at $t+1$, $M_t n_e$ units of labor are used in period t. These new firms have productivity drawn from density $g(z)$.

Existing firms in this economy solve a static profit maximization problem, choosing employment $l_t(z)$ to solve

$$\max_{l} z^{(1-\nu)} l^{\nu} - w_t l$$

Let the maximized profits at t (given wage w_t) be denoted $\pi_t(z)$.

There is free entry into the business of starting a new firm. We let p_t denote the price of a unit of consumption at t relative to consumption at $t = 0$. There are zero profits to entry if and only if

$$n_e w_t = \sum_{k=1}^{\infty} \frac{p_{t+k}}{p_t} (1-\delta)^{k-1} \int_z \pi_{t+k}(z) g(z) dz$$

We require that there be zero profits to entry at t if $M_t > 0$ and non-positive profits to entry at t if $M_t = 0$.

The evolution of the number of firms is given by

$$N_{t+1} = (1-\delta) N_t + M_t$$

and the evolution of the distribution of productivities across firms is given implicitly by

$$f_{t+1}(z) N_{t+1} = (1-\delta) f_t(z) N_t + g(z) M_t$$

Let $l_t(z)$ denote the hiring decision of a firm with productivity z at t. Aggregate output is given by

$$y_t = N_t \int_z z^{(1-\nu)} l_t(z)^\nu f_t(z) dz$$

Goods market clearing requires $c_t = y_t$.

Labor market clearing requires that

$$N_t \int_z l_t(z) f_t(z) dz + M_t n_e = 1$$

An equilibrium in this economy is a collection of sequences $\{c_t, y_t, l_t(z), N_{t+1}, M_t\}$, prices $\{p_t, w_t\}_{t=0}^\infty$, and firm profits $\{\pi_t(z)\}$ such that the representative household is choosing $\{c_t\}$ to maximize its utility subject to a date zero budget constraint

$$\sum_{t=0}^\infty p_t \left[w_t + N_t \int_z \pi_t(z) f_t(z) dz - c_t \right] \geq 0,$$

firms are choosing labor $l_t(z)$ to maximize profits, there are zero profits to entry at t if $M_t > 0$ and non-positive profits if $M_t = 0$, and goods and labor markets clear.

Part A: Show how to reduce the problem of finding equilibrium to one of finding aggregate sequences $\{c_t, y_t, N_{t+1}, M_t\}$, and prices $\{p_t, w_t\}_{t=0}^\infty$. To do so, you will need to aggregate across firms to construct a measure of aggregate productivity Z_t and describe the evolution of this productivity variable over time.

Part B: Now assume that firms must pay a value added tax τ so that their revenues are given by $(1-\tau) z^{(1-\nu)} l_t(z)^\nu$. Assume that this tax revenue is rebated back to the household lump sum. How does this tax affect the equilibrium allocation (describe as much as you can).

Question 5

Consider an economy with a measure one continuum of households. The households evaluate a sequence of consumption and labor supply choices allocations and labor supply choices $\{c_t, n_t\}_{t=0}^{\infty}$ according to the utility functional $U = \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - v\left(n_t\right)\right]$.

Each period, households receive an idiosyncratic shock $\theta \in \{\theta_L, \theta_H\}$, which characterizes the efficiency of its labor supply; a household whose labor supply is n_t provides θn_t efficiency units of labor. θ evolves according to a first-order Markov process with transition probabilities $\pi\left(\theta'|\theta\right)$. The fraction of households who start in state θ_L at date 0 is λ_0 (you may assume that λ_0 corresponds to the steady-state fraction of low types).

In addition, there is a continuum of firms, which produce output of the consumption good c using *efficiency units of labor* y according to the technology $c = y$.

Part A: Flexible wages: Characterize the optimal allocations, if the planner can fully enforce all transfers to and from households. Describe a decentralization of this allocation in a competitive equilibrium.

Part B: Sticky wages: Suppose now that households can renegociate their wages and hours only every 2 periods (and 1/2 the households negociate each period). Formally, suppose that in each period, firms offer contracts, i.e. pairs (w, n) of wages and hours, that are contingent on a households current efficiency state, and households then accept the most attractive offer.

Define the equilibrium with sticky wages, and offer a characterization of the steady-state equilibrium. What distortions result from wage stickyness in equilibrium? How does your answer depend on the availability of complete insurance markets against idiosyncratic shocks?

Question 6

Consider the following discrete-time, infinite horizon risk-sharing problem between a risk-averse agent and a risk neutral principal, with limited commitment. The planner can commit to future transfers, but the agent can't: At each date, the agent has the option to walk away from the risk-sharing agreement, in which case his subsequent consumption equals his endowment (autarky).

The agent receives a constant endowment $y = 1$ in each period, and ranks consumption plans according to $\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right\}$, where $\beta \in (0,1)$, and $u(\cdot)$ is increasing, concave, bounded and twice differentiable. The principal is risk-neutral but faces external interest rate shocks: he ranks the same sequence according to $\mathbb{E}\left\{\sum_{t=0}^{\infty} \lambda\left(s^t\right)\left(1 - c_t\right)\right\}$, where the discount factor $R\left(s^t\right) = \lambda\left(s^{t+1}\right)/\lambda\left(s^t\right)$ is iid over time and distributed according to

$$R\left(s^t\right) = \begin{cases} \beta^{-1} \text{ w.p. } \pi \\ R < \beta^{-1} \text{ w.p. } 1-\pi \end{cases}$$

which fully describes the event tree along with the corresponding probabilities.

Set up the planning problem associated with this risk-sharing problem, and characterize its solution. What are the implications of external interest rate shocks for optimal consumption smoothing with limited commitment?