June 24, 2004

UCLA Department of Economics

First-Year Core Comprehensive Examination in MACROECONOMICS

Spring 2004

Answer all questions offered. Each question has equal weight.

Answer each question in a separate answer book. Put the question number on the outside of each book.

You have **4 hours**

Spring, 2004 Macro Comprehensive Exam – Question 1

A social planner in a two agent economy faces the problem

$$Max \lambda U(C^1) + (1-\lambda)U(C^2)$$

such that

$$k_{t+1} = f(k_t)s_t - c_t^1 - c_t^2, \quad t = 1,..\infty,$$
$$k_1 = \bar{k}.$$

The term s_t is a stochastic shock with mean $\bar{s}$. Each period, s_t can take the values

$$s_t = \begin{cases} s_a \text{ with probability } p \\ s_b \text{ with probability } 1-p. \end{cases}$$

Define the sets S^t as

$$S = \{s_a, s_b\},$$
$$S^1 = S,$$
$$S^t = S \times S^{t-1},$$

and let the history s^t be given by

$$s^1 = s_1$$
$$s^t = \{s_t, s^{t-1}\}.$$

The social planner chooses functions $C^1 = \{c^1(s^t)\}_{t=1}^{\infty}, C^2 = \{c^2(s^t)\}_{t=1}^{\infty}, K = \{k(s^{t-1})\}_{t=2}^{\infty}$ by allocating resources in each period between next period's capital stock and current consumption of each agent. The utility of each agent is described by the function,

$$U(C^i) = \sum_{s=t}^{\infty} E\left[\frac{\beta^{s-t}(c_t^i)^{1-\rho}}{1-\rho}\right],$$

where E is the expectations operator. The technology takes the form

$$f(k) = Ak^{\alpha}.$$

1. Assume that the social planner solves the sequence problem in period 1. How many values of consumption must he choose for agent i in period 1? How many in period 2? How many in period n?
2. Write down a set of necessary and sufficient conditions that fully characterizes the solution to the social planning problem. How do you know that this solution is unique?
3. Consider the alternative social planning problem in which $s_t = \bar{s}$ for all t. Find an analytic solution for this alternative problem for the steady state variables $\bar{c}^1$, $\bar{c}^2$ and $\bar{k}$ as functions of the fundamental parameters A, α, ρ and β.
4. What is meant by the term "complete markets"? Describe carefully a set of securities that would be sufficient to in order for a competitive economy to decentralize the social planner's problem by means of markets.
5. Write down the problem solved by each of the consumers in your market economy. In the

context of this problem, explain what is meant by the 'no Ponzi scheme' condition? Is this the same as the transversality condition? If not, how do they differ?

6. The competitive economy is described by two more equations than the social planning solution. What are these equations and how are they related to the social welfare weight λ?
7. What is meant by a 'sunspot equilibrium'? Can this economy display sunspots? If so- explain how you would construct them. If not, explain why not.

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An Economy where Experience Matters

Question 2

(a) Consider an economy with mass one of identical consumers whose preferences are defined by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - n_t\right],$$

where c_t is consumption, n_t is labor supply, and the parameter β satisfies $0 < \beta < 1$. Labor supply is restricted to lie in the interval (0,1). There is a competitive industry which operates the production technology:

$$y_t = k_t^{\alpha} N_t^{1-\alpha},$$

where k_t is capital, N_t is aggregate labor supply, and the parameter α satisfies $0 < \alpha < 1$. Capital depreciates completely every period, i.e., the law of motion for capital is $k_{t+1} = i_t$, where i_t is investment. The initial level of capital is given by k_0. In this version of the economy, the market-clearing condition for labor is given by $N_t = n_t$.

- Provide a Bellman equation for the social planning problem.
- Define a sequence-of-markets equilibrium for this economy.
- Find the steady-state levels of capital and labor supply.

(b) So far, we have implicitly assumed that the quality of labor supplied by the households is always the same. In this part of the question, we will assume that the quality of labor depends on prior labor market experience. The experience of a given household is given by e_t. The stock of experience depreciates at rate δ, but it can be increased through work. The law of motion for experience is given by:

$$e_{t+1} = (1-\delta)e_t + n_t^2.$$

Notice that experience is convex in labor supply, that is, experience increases faster the longer a given person works.

- Provide a Bellman equation for the problem solved by a benevolent social planner under the restriction that all households work the same amount of time (in this case, aggregate labor supply is given by $N_t = e_t n_t$).
- Now consider an alternative arrangement with lotteries such that, ex post, some people work full-time ($n_t = 1$), while others do not work at all ($n_t = 0$). Explain whether it would be possible to improve upon the optimal allocation where everybody is treated identically with such an arrangement. Can you guess what the optimal outcome would look like, i.e., would consumption vary across workers and non-workers, and would the same people be working in every period?

3. Consider the following environment. Assume there is no capital and that output is produced solely with labor. The realized output level depends upon the productivity shock z. We will also assume that government spending is stochastic as well. Let s^t which will denote the state in period t be given by

$$s^t = \{z_j, g_j\}_{j=0}^t.$$

We will assume that the initial state is not stochastic. Let $\mu(s^t)$ denote the probability of s^t and let $\mu(s^t|s^{t-1})$ denote the conditional probability. The resource constraint for this economy is given by

$$c(s^t) + g(s^t) = z(s^t)l(s^t).$$

The government's budget constraint is given by

$$g(s^t) + R(s^t)b(s^{t-1}) \leq \tau(s^t)w(s^t)l(s^t) + b(s^t),$$

where R denote the gross return on government debt. Household preferences are given by

$$\sum_t \sum_{s^t} \beta^t u(c(s^t), l(s^t))\mu(s^t),$$

and their budget constraint by

$$c(s^t) + b(s^t) \leq (1 - \tau(s^t))w(s^t)l(s^t) + R(s^t)b(s^{t-1})$$

A) Construct the Ramsey problem and characterize the efficient allocation. Discuss how you would compute this solution if you were given an explicit set of preferences and stochastic process for s_t that was first-order Markov.

B) Is this problem time-consistent (i.e. if we allowed the planner to consider resolving the problem in the second period would he want to change his solution) ?

Question 4 - Technological Progress and Development

There are 3 consumption goods: an agricultural good (f_t), services (s_t) and a manufactured good (m_t). Preferences are given by:

The consumer's maximization problem is given by:

$$\max \sum \beta^t \{\ln(f_t) + \ln(s_t) + m_t + \phi \ln(1 - l_f - l_m - l_s)\}$$

The technologies are

$$M_t = A_t L_{Mt},\ F_t = A_t L_{Ft},\ S_t = A_t L_{St}$$

Technological progress evolves as:

$$A_t = A_0(1+\gamma)^t, \gamma > 0, t = 1,2,\ldots$$

(A) Write this as a social planning problem, and solve for the planner's first order conditions. If you decentralized this problem as a competitive equilibrium, would all the prices be the same, or would they be different? Explain your answer. (You can do this without constructing the equilibrium).

(B) Show that there exists a level A^* such that for $A_t \leq A^*$, the economy will produce only agricultural goods and services. How much labor is employed for each of these goods in this case? Show that labor supply is constant in the interval $A \in (0, A^*]$.

(C) Show that for $A_t > A^*$, all goods are produced. Describe what happens to the fraction of employment devoted to farm goods and services asymptotically. What happens to total labor supply asymptotically?

(D) Suppose a tax is levied on the production of services. That is, for every unit of services produced, the government confiscates τ units, and redistributes the proceeds lump-sum. How much will the relative price of services rise? (Again, you can answer this question without constructing the full equilibrium). Does this affect the value of A^*?