UCLA, Department of Economics
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# Comprehensive Exam in Macroeconomics

**Instructions:** This exam consists of six questions. You should answer **5 out of 6** questions. Each question should take about the same amount of time (30 minutes).

Answer each question in a **separate bluebook.** Be sure to put the question number on the outside of each bluebook.

**Question 1**

Consider an artificial economy the equilibrium of which solves the following social planning problem:

$$\max \sum_{t=0}^{\infty} \beta^t N_t \frac{\left(c_t^{\alpha} l_t^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma}, \quad 0<\beta<1,\ 0<\alpha<1,\ \sigma>0$$

subject to

$$N_t(C_t + I_t) = (N_t K_t)^{\theta}(\gamma^t N_t h_t)^{1-\theta}, \quad \gamma \geq 1,\ 0<\theta<1$$
$$0 \leq h_t \leq 1$$
$$N_{t+1}K_{t+1} = (1-\delta)N_t K_t + N_t I_t, \quad 0<\delta<1$$
$$N_{t+1} = \eta N_t, \quad \eta \geq 1$$

and $K_0$ and $N_0 = 1$ are given.

$N_t$ = population

$C_t$ = consumption (per capita)

$I_t$ = investment (per capita)

$h_t$ = hours worked (per capita)

$K_t$ = capital stock (per capita)

A. Write this planners problem as a dynamic programming problem.

B. Characterize for the balanced growth path of this economy. What is the growth rate of each of the endogenous variables along this balanced growth path?

C. Discuss how one might calibrate a quarterly version of this economy to the following features of the post Korean War U.S. economy. Be as specific as possible without necessarily finding a numerical value for each parameter.

   (i) The average annual growth rate of per capita real output is 1.4 percent.

   (ii) The average annual growth rate of the population is 1.5 percent.

   (iii) The average fraction of total income that is paid to owners of capital is .4.

   (iv) The average investment to output ratio is .25.

   (v) The average capital to output ratio is 3.5.

   (vi) Individuals spend 31% of their substitutable time working.

   (vii) The value of $\sigma$ is 1.5.

**Question 2**

Consider a world economy made up of two infinitely lived countries (households) i=1,2 of equal population size. The countries are identical in all respects except for the rates of time preference $0<\beta_2<\beta_1<1$. They share a common constant-returns technology $f(k)=k^{\alpha}$ with no technical progress, depreciation rate $\delta \in (0,1)$ a flow utility function $u(c)=\log c$ and a constant labor supply equal to 1 each period. Labor is not mobile across national boundaries.

(a) Suppose existing laws prohibit capital movements altogether. Also let $\delta=.08,\quad 1/\beta_1=1.02,\quad 1/\beta_2=1.10$ and $\alpha=1/3$. Find per capita incomes in the steady state for each country.

(b) Assume that laws restricting capital imports and exports are suddenly and unexpectedly removed. Describe per capita GDP and GNP for each country in the steady state.

(c) Does capital mobility improve the per capita GNP of the least patient country i=2? Does it improve its share of world output? Explain your answer.

## Question 3

Consider the following environment in which output in each period is produced using capital and labor. Households own the capital stock, and rent capital and labor to the firms. The government uses flat rate labor income and capital income taxes to finance a constant stream of government expenditures. We will assume that there exists a representative household. We will also assume that the firms' production function exhibits constant-returns-to-scale and hence that there exists a representative firm.

In particular, the environment has the following features:

**Government's budget constraint:**

$$g+(1+r_t)b_t = b_{t+1}+\tau_{l,t}w_t l_t+\tau_{k,t}(q_t-\delta)k_t .$$

**Household's problem:**

$$\max \sum_t \beta^t u(c_t, l_t)$$

subject to

$$(1-\tau_{l,t})w_t l_t+(1-\tau_{k,t})(q_t-\delta)k_t+(1+r_t)b_t = c_t+k_{t+1}-k_t+b_{t+1}$$

$k_{t+1}\geq 0$ and $k_0$, $b_0$, $r_0$ are given.

**Firm's problem:**

$$\max\left[f(k_t,l_t)-q_t k_t-w_t l_t\right]$$

**Resource contraint:**

$$c_t+k_{t+1}-(1-\delta)k_t+g = f(k_t,l_t)$$

A. Derive the optimality conditions that characterize solutions to the household's and the firm's problems. Define a competitive equilibrium. Note that this definition must include the sequence of taxes.

B. Construct the implementability condition and show that any allocation that satisfies this condition along with the resource constraint can be decentralized as a competitive equilibrium.

C. Construct the government's optimal policy problem and the conditions that characterize a solution to it. Show that if the solution converges to a steady state, then capital taxes must equal zero in the steady state.

**Question 4**

Consider the following simple money in the utility function model. Preferences are given by

$$\sum_t \beta^t \left[ u(c_t) + v(m_{t-1}/p_t) \right],$$

where $u$ and $v$ are assumed to be $C^2$, increasing and strictly concave, with $u'(0) = \infty$. This is a simple endowment economy in which households receive $y$ units of consumption each period $t = 1, 2, \ldots$, and are endowed with $M_0$ units of money and receive a monetary transfer from the government in the amount of $T_t$ each period. The household budget constraint in all periods $t \geq 1$ is given by

$$p_t c_t + m_t + b_t \leq m_{t-1} + R_{t-1} b_{t-1} + p_t y + T_t$$

with $m_0 = M_0$, $R_0 b_0 = 0$ given and the nonnegativity restriction that $m_t \geq 0$. The households also face a no-Ponzi scheme constraint which requires that they always be in a position to pay off their debts, or

$$b_t + m_{t-1} + R_{t-1} b_{t-1} + p_t y + T_t + \sum_{j=1}^{\infty} \frac{(p_{t+j} y + T_t)}{\prod_{s=0}^{j-1} R_{t+s}} \geq 0 \quad \forall t \geq 1.$$

A. Assume that $T_t = gM_{t-1}$; that is, transfers are such that the money supply grows at a constant rate $g \geq \beta$. Construct the representative household's problem and use it to characterize an equilibrium.

B. Assume that there is a satiation level of real balances such that for all $m_t/p_t \geq \bar{M}$, $v'(m_t/p_t) = 0$. What are the weakest set of conditions that a constant money growth rate rule must satisfy such that there exists an efficient equilibrium? Be sure to prove that if these conditions hold, an equilibrium with a zero nominal interest rate exists.

C. Do there exist bubble hyperinflations equilibria as well for money supply rules that satisfy your conditions in (B)? Be sure to explain why you think that your answer is true.

**Question 5**

The maximization problem for the representative household is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{c_{mt}^{\alpha} c_{ht}^{1-\alpha} - B(l_{ht} + l_{mt})\}$$

where $c_{mt}$ is consumption of the market good, $c_{ht}$ is consumption of the home good, $l_m$ is time spent working in the market, and $l_h$ is time spent working at home.

The resource constraints are:

$$\gamma_t z_{mt} l_{mt} \geq c_{mt}$$

$$\gamma_t z_{ht} l_{ht} \geq c_{ht}$$

$$1 \geq l_{mt} + l_{ht}$$

The parameter $\gamma$ is a common technology shock that follows a standard two state Markov process, with the shock taking either a high or low value. The parameters $z_m$ and $z_h$ are technology shocks that are specific to the market technology, and the home technology, respectively. These shocks also governed by two state markov processes that take either high or low values. These three Markov processes are independent and the transition probabilities for both the high state and low state for all three processes is equal to .5, irrespective of the current state.

(1) Write this as a social planner's problem, and solve for the planner's first order necessary conditions.

(2) How does total labor input ($l_{ht} + l_{mt}$) respond to the aggregate technology shock $\gamma$? Explain.

(3) How does the relative allocation of labor between sectors respond to sector specific shocks? Explain.

**Question 6**

Assume that you have data on the returns earned on two assets, denoted $R^1$ and $R^2$ respectively. Assume that, in the data that you have, the difference in the rate of return on these two assets, $R_t^1 - R_t^2$, has a mean of .08 and a standard deviation of 0.16. Use the asset pricing equation,

$$1 = E_t \beta \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1}^i \text{ for } i = 1, 2,$$

to argue that theory predicts that the standard deviation of

$$\beta \frac{u'(c_{t+1})}{u'(c_t)}$$

must be at least half the mean of this variable. Explain how one might use data on consumption to assess whether or not this asset pricing theory can account for the data on asset prices.