

## Macroeconomic Theory Core Exam

### Instructions:

You have four hours to complete this exam.  
There are three parts. Each has equal weight.

**Answer each part in a separate blue book.**

### PART I

**Answer in separate blue book**

Describe the evolution of public debt in a growing economy with the following structure:

- i. zero initial public debt ( $b_0 = 0$ );
- ii. given initial stock of capital ( $k_0 > 0$ );
- iii. no taxes;
- iv. an exogenous positive sequence of per capita government purchases ( $g_t$ ) financed by deficit spending, and representing pure waste;
- v. a fiscal policy that specifies constant deficit-to-GDP ratio

$$g_t/f(k_t) = q > 0 \quad \forall t$$

where  $q$  is a fiscal policy parameter.

- (a) Define a "sustainable fiscal policy" in general.
- (b) Is the fiscal policy described above sustainable according to the optimum growth model?
- (c) What does the overlapping generations model say about this question? Does the OLG model put an upper bound on the policy parameter  $q$ ?

**PART II**  
**Answer in separate blue book**

Consider a standard Real Business Cycle model in which a continuum of identical households have preferences given by  $E \sum_{t=0}^{\infty} \beta^t (\log c_t + A \log(1 - h_t))$ , where  $c_t$  is consumption,  $h_t$  is hours worked, and  $0 < \beta < 1$ . The technology is given by a Cobb-Douglas production function with capital and labor as inputs:  $y_t = e^{z_t} k_t^\theta h_t^{1-\theta}$ . The variable  $z_t$  is a technology shock that follows a two-state Markov chain with transition matrix  $P$ . The law of motion for the capital stock is  $k_{t+1} = (1 - \delta)k_t + i_t$ , where  $i_t$  is investment and  $0 \leq \delta \leq 1$ . The resource constraint is  $c_t + i_t \leq y_t$ .

- A. Discuss the choice of utility function and production function in this economy. That is, what considerations have led researchers to use these particular functional forms (or perhaps similar ones). Be brief.
- B. Describe how one can use features of U.S. time series to calibrate this economy. That is, describe a mapping between a set of statistics computed from U.S. data (list the statistics, you don't need to give actual numbers) and values for  $\beta$ ,  $A$ ,  $\theta$ , and  $\delta$ . How might one go about calibrating the Markov chain for  $z$ ?
- C. Suppose now that labor is indivisible and that  $h_t \in \{0, \hat{h}\}$ . In addition, assume that there is a market for employment lotteries. Define a *Recursive Competitive Equilibrium* for this economy, including markets for goods and capital services in addition to the employment lotteries. Be careful in stating the problems solved by households and firms.
- D. It has been shown that a real business cycle model like the one described here displays larger fluctuations in hours worked relative to productivity than a model with divisible labor. Provide some intuition for this result. Also, describe why this result is important within the context of the real business cycle literature.
- E. Suppose now, that in addition to the markets in part C, there is a market for one period ahead contingent claims to output (contingent on the value of the technology shock). Define a recursive competitive equilibrium in this case.

**PART III**  
**Answer in separate bluebook**

A representative household maximizes:

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ (\alpha c_{1t}^\sigma + (1-\alpha)c_{2t}^\sigma)^{\frac{1}{\sigma}} - Bn_t \right\},$$

The resource constraints are:

$$A_{1t}K_{1t}^\theta N_{1t}^{1-\theta} = C_{1t} + K_{2t+1}$$

$$A_{2t}K_{2t}^\theta N_{2t}^{1-\theta} = C_{2t} + K_{1t+1}$$

$$1 \geq N_{1t} + N_{2t} + L_t$$

The stochastic processes for the shocks are:

$$A_{1t} = 1 + \varepsilon_{1t}$$

$$A_{2t} = 1 + \varepsilon_{2t}$$

The random variables  $\varepsilon_1$  and  $\varepsilon_2$  are identically and independently distributed uniform random variables with support between (.9,1.1).

- A. Write this problem in Dynamic Programming form as a social planners problem, and find the first-order necessary conditions.
- B. Characterize the steady state of the model.
- C. Define a recursive competitive equilibrium for this economy, and solve for the competitive equilibrium allocations and prices. Why does the planner's optimum coincide with the competitive equilibrium allocations?
- D. Define the market value of output from the 2 sectors, and denote that market value  $Y$ . (Assume that the market value is calculated according to steady state prices). Use the equilibrium prices and allocations to solve for:

$$\frac{\partial \ln(Y_t)}{\partial \ln(A_{1t})}$$

How does the relative size of the sector ( $\alpha$ ) and the substitution elasticity between the two goods determine the macroeconomic impact of a sectoral shock?