QUALIFYING EXAM IN MACROECONOMICS

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Instructions: You have four hours to complete this exam. There are three parts. Each has equal weight. **Answer each part in a separate blue book.**

Part One (*Blue Book Number 1*)

This question is about government deficits and public debt in the life cycle, or OLG, model of pure exchange. Consider an OLG model of pure exchange with two-period lived identical households, in which population is constant, the endowment vector is (e_1, e_2), and the utility function is $v(c_1, c_2) = c_1 + \beta c_2$, $\beta > 0$, for each agent.

(a) Assume no government purchases, taxes or debt. Describe individual consumption-saving decisions as a function of the interest factor R. For what values of the parameters (β, e_1, e_2) is this economy of the Samuelson type?

(b) Under the same assumption as in (a), compute the equilibrium rate of interest as a function of the parameters.

(c) Assume now that this economy is of the Samuelson type, government purchases and taxes are zero, public debt is positive and is rolled over each period. Describe the evolution of the public debt over time and all the steady states of this economy. [HINT: Write down the first-order difference equation appropriate for this economy. Use per capita debt as your state variable.]

(d) What is the maximal sustainable stock of per capita public debt in part (c)?

(e) Repeat part (c) assuming now that the government runs a primary budget deficit $g > 0$. Assume all taxes are zero. Again write down a first-order difference equation that describes the evolution of per capita public debt.

((f) What is the maximum sustainable deficit in part (e)? What is the maximum sustainable initial public debt?

Part Two (*Blue Book Number 2*)

Consider an economy with a large number of infinitely lived identical households with preferences given by,

$$\sum_{t=0}^{\infty} \beta^t \log c_t \,.$$

Each household is endowed with k_0 units of capital in period 0 and 1 unit of labor each period. The number of households in period t is N_t, where $N_{t+1} = \eta N_t$, $\eta > 1$.

We will consider two alternative technologies for this economy:

Technology 1 $\quad Y_t = \gamma^t K_t^{\theta} N_t^{1-\theta}$

Technology 2 $\quad Y_t = \gamma^t K_t^{\mu} N_t^{\phi} L_t^{1-\mu-\phi}$

In these technologies, $\gamma > 1$ is the rate of exogenous total factor productivity growth, K_t is total (<u>not</u> *per capita*) capital, Y_t is total output, and L_t is the total stock of land. Land is assumed to be a fixed factor, it cannot be produced and does not depreciate. To simplify with out loss of generality, assume that $L_t = 1$ for all t.

The resource constraint, assuming 100 percent depreciation of capital each period, is given by

$$N_t c_t + K_{t+1} \leq Y_t \,.$$

A) Suppose that the only technology available is the first one.

   i) Formulate, as a dynamic programming problem, the social planner's problem that weights all individuals utility equally. That is, the planner weights utility in period t by the number of identical agents alive in that period.

   ii) Characterize the balanced growth path of this economy. ("Characterize" means that you must derive a set of equations that determines all endogenous variables along this path. You do not need to solve these equations.) Solve explicitly for the growth rate of per capita consumption (c_t) along this path.

B) Repeat part A) using the second technology in place of the first.

C) Compare how the rate population growth η affects the rate of per capita growth in the two cases. Provide intuition for your findings.

D) Formulate the social planner's dynamic programming problem for an economy in which both technologies are available. Will the planner necessarily employ the second technology in such an economy? Explain.

Part Three (*Blue Book Number 3*)

Consider the following model in which there exist a representative family that maximizes the utility function:

$$(1) \qquad U = \sum_{t=1}^{\infty} \beta^{t-1} \frac{C_t^{1-\rho}}{1-\rho},$$

where ρ is a positive parameter, β is the discount factor and C_t is consumption. Output is produced from the technology:

$$(2) \qquad Y_t = A\left(\frac{M_t}{P_t}\right)^{\alpha},$$

where α is a positive parameter between zero and one, A is a positive parameter, M_t is the nominal quantity of money held between periods t and $t+1$ and P_t is the price of commodities in terms of money. All output is consumed. The family holds zero bonds and $\overline{M}$ units of money at the beginning of period 1.

The money supply is increased each period according to the rule:

$$(3) \qquad M_{t+1} = \mu M_t, \qquad \mu > 1, \qquad M_0 = \overline{M}$$

and all new money is distributed to the representative family as a lump sum transfer, denoted T_t, measured in units of money. The family may choose to hold its wealth in the form of money, or government bonds, B_t, where B_t represents bonds held between periods t and $t+1$. Bonds are in zero net supply. The nominal interest rate on a bond that is held between periods t and $t+1$ is denoted i_t. Let the present value of a period s dollar in period t be denoted by Q_t^s.

A. What is meant by a present value price? The term Q_t^t is defined to be identically equal to 1. Write an expression that defines Q_t^s as a function of the sequence of interest rates $\{i_j\}_{j=t}^{s}$ for $s > t$.

B. Assume that the family owns a representative firm and that in each period sells output, buys consumption goods and accumulates money and bonds in a sequence of markets. Write down the budget constraint faced by the family/firm in each of these markets.

C. Write down a single constraint for the family which constrains its expenditues and consumption plans for periods 1 through T. [HINT: this expression should involve terms in $\frac{M_t}{P_t}$, C_t, T_t, i_t, Q_1^t and Y_t, for $t = 1,\ldots T$].

D. What is meant by a Ponzi scheme? Write down a constraint that prevents the family from running a Ponzi scheme.

E. Write down a single infinite horizon budget constraint for the family. [HINT: this expression should involve terms in $\frac{M_t}{P_t}$, C_t, T_t, i_t, Q_1^t and Y_t, for $t = 1,\dots\infty$].

F. Using equation (2) and your answer to part (B) rewrite the utility function in terms of real balances $\frac{M_t}{P_t}$ and real bonds $\frac{B_t}{P_t}$. [Assume that the period budget constraint holds with equality.] Find a first order condition for the choice of real balances when the family firm maximizes the utility function you have derived written in terms of money and bonds.

G. Find a difference equation in real balances that must hold in a competitive equilibrium. [Hint: Use the equilibrium conditions, $C_t=Y_t$ and eliminate P_t from the problem using the money supply rule].

H. Find an expression for the unique steady state equilibrium value of real balances as a function of the parameters of the problem. Does an increase in the money growth rate raise or lower real balances? Provide intuition as to why this occurs.

I. What is meant by indeterminacy of equilibrium? Explain in words what condition is necessary for this steady state equilibrium to be indeterminate.